and AE can be drawn, cutting off triangles ADC and AEC, the one less, and the other greater, than ABC, but neither of them differing from the triangle ABC by an area so great as a given area; while at the same time the difference between the sum of the angles of the triangle ABC and the sum of the angles of either of the triangles, ACD, ACE, is less than any given angle.

If the hypothesis be made that the angles of a plane triangle are together (see Cor. Prop. II.) equal to two right angles, the problem can be effected by the methods which Euclid describes.

We only need, therefore, to show how it can be performed on the hypothesis that the angles of a plane triangle are not equal to two right angles. Bisect CG in F; and join AF. The triangles ABC and ACF have a comon side AC. Therefore (Cor. 4) the area of the one will (on the hypothesis on which we are now proceeding) be less than, equal to, or greater than, the area of the other, according as the sum of the angles of the former is greater than, equal to, or less than, the sum of the angles of the latter. Now we can find the sum of the angles of each by construction. Therefore we can tell whether the triangle ACF is less than, equal to, or greater than, the triangle ABC. Should the triangle ACF be greater than the triangle ABC, we may repeat the construction; bisecting CF, and drawing a line from A to the point of section. By repeating this construction sufficiently often, the base (such as CD) of the triangle (such as ACD) ultimately obtained will become less than any assignable line; and hence the area of the triangle will become (Prop. III.) less than any assignable area, and consequently less than the triangle ABC. Let ACD, the triangle obtained by bisecting CE, and joining AD, be less than the triangle ABC; the triangle AEC being greater than ABC. Bisect DE in the point *t*; and join A*t*. Find, as above, whether the triangle AC*t* is less or greater than the triangle ABC, or equal to it. Should it be greater, the triangle ABC lies between the limits, ACD and AC*t*; but should it be less, the triangle ABC lies between the limits AC*t* and ACE. And so on. Ultimately we obtain two limits, which we may suppose to be represented by the triangles ACD and ACE, between which the triangle ABC lies, the base DE of the triangle ADE, which is the difference of the limits, being made as small as we please. Therefore (Prop. III.) the area of the triangle ADE becomes ultimately indefinitely small; so that each of the triangles ACD and ACE becomes indefinitely near in area to the triangle ABC.