occur in pairs, the four roots of the equation must be

$$\tfrac12\sqrt3+\sqrt{-1},\quad \tfrac12\sqrt3-\sqrt{-1}$$
$$-\tfrac12\sqrt3+\sqrt{-1},\quad -\tfrac12\sqrt3-\sqrt{-1}$$

hence the required equation will be formed by equating to o the product of the four quantities

$$x-\tfrac12\sqrt3+\sqrt{-1},\quad x-\tfrac12\sqrt3-\sqrt{-1}$$
$$x+\tfrac12\sqrt3+\sqrt{-1},\quad x+\tfrac12\sqrt3-\sqrt{-1}$$

This gives $16x^4+8x^2+49=o$.

30. (*a*). If we go on dividing by 19, the second dividend is 50, and consequently the result obtained will be half that produced by dividing 19 into 100; we may, therefore, obtain the remaining digits in the quotient by dividing the part already obtained by 2, placing the figures thus obtained to the right of those already found, and then using them as part of the dividend; thus we get

$$.\dot{0}5263157894736842\dot{1}.$$

(*b*). Since this repeater extends to its full limit of 18 digits, it follows that all the remainders, from 1 to 18 inclusive must have occurred in the course of the division; and when any remainder occurs, the subsequent division must give the digits in the decimal corresponding to the fraction having this remainder for numerator. Thus the fourth remainder is 12, and the division thereafter gives the decimal for $\frac{12}{19}$, which is therefore $.\dot{6}31$, &c.

If therefore we wish to find the repeating decimal corresponding to any fraction with 19 for denominator, say $\frac{7}{19}$ we have only to divide far enough to ascertain what digit in the value of $\frac{1}{19}$ is the first digit in the value of $\frac{7}{19}$. Thus $\frac{7}{19}$ gives .36, and we have simply to refer to the decimal for $\frac{1}{19}$ to obtain the remaining figures; the value of $\frac{7}{19}$ is thus found to be

$$.\dot{3}6842105263157894\dot{7}$$

31. $$(b^2-c^2)x^2+(c^2-a^2)y^2+(a^2-b^2)z^2$$
$$+2c(b-a)xy+2a(c-b)yz+2b(a-c)zx$$
$$=(bx+cy+az)^2-(cx+ay+bz)^2$$
$$=(bx+cy+az-cx-ay-bz)\times$$
$$(bx+cy+az+cx+ay+bz)$$

This last factor

$$=(b+c)x+(c+a)y+(a+b)z$$
$$=(b+c+a)x-ax+(c+a+b)y-by$$
$$+(a+b+c)z-cz$$
$$=(a+b+c)(x+y+z)-(ax+by+cz)$$
$$=0 \text{ if } \begin{cases} ax+by+cz=0 \\ a+b+c=0 \end{cases}$$

so that the equation is satisfied if these conditions hold.

32. Let k denote the sum originally held by the rth person, then after the first distribution he will have $2k$

after the second, $4k$
" " third, $8k$
&c., &c.
after the $(r-1)^{th}$, $2^{r-1}k$

At this point the others together must have

$$s-2^{r-1}k$$

which is therefore the sum he gives away at the rth distribution. After the rth distribution, therefore, he has

$$2^{r-1}k-(s-2^{r-1}k)=2^rk-s$$

after the $(r-1)^{th}$ he has $2(2^rk-s)$
and after the n^{th} distribution, he has

$$2^{n-r}(2^rk-s)=2^nk-2^{n-r}s$$

Hence he has gained or lost according as

$$2^nk-2^{n-r}s > \text{or} < k$$
i.e. as $(2^n-1)k >$ or $< 2^{n-r}s$
i.e. as $k >$ or $< \dfrac{2^{n-r}}{2^n-1}s$

33. $$\frac{2}{\sqrt{(x-a)}}+\frac{1}{\sqrt{(x-b)}}=\frac{1}{\sqrt{(x-c)}}$$
$$\therefore\ 2\sqrt{(x-b)}\sqrt{(x-c)}+\sqrt{(x-c)}\sqrt{(x-a)}$$
$$=\sqrt{(x-a)}\sqrt{(x-b)}$$

square and transpose, then

$$4(x-b)(x-c)+(x-c)(x-a)-(x-a)(x-b)$$
$$=-4(x-c)\sqrt{(x-a)}\sqrt{(x-b)}$$

square and transpose again, then

(1). $$o=16(x-b)^2(x-c)^2+(x-c)^2(x-a)^2$$
$$+(x-a)^2(x-b)^2-8(x-c)^2(x-a)(x-b)$$
$$-8(x-b)^2(x-c)(x-a)-2(x-a)^2$$
$$(x-b)(x-c)$$

The same process with

$$\frac{2}{\sqrt a}+\frac{1}{\sqrt b}=\frac{1}{\sqrt c} \text{ gives}$$

(2). $$16b^2c^2+c^2a^2+a^2b^2-8c^2ab-8b^2ca-$$
$$2a^2bc=0$$

In (1) x^4 disappears, and on subtracting (2)