$ABC$ be a triangle circumscribed to an ellipse, the eccentric angles of the points of contact being $\alpha, \beta, \gamma$; let $P, Q, R$ be the middle points of its sides, and $O$ as before centre of the ellipse. If $x_1y_1, x_2y_2, x_3y_3$ be the co-ordinates of $A, B, C$, then

$$8(QOR) = \begin{vmatrix} -1, x_1, y_1 \\ 1, x_2, y_2 \\ 1, x_3, y_3 \end{vmatrix}$$

and $\dfrac{x_1}{a} = \dfrac{\cos\dfrac{\beta+\gamma}{2}}{\cos\dfrac{\beta-\gamma}{2}}$, $\dfrac{y_1}{b} = \dfrac{\sin\dfrac{\beta+\gamma}{2}}{\cos\dfrac{\beta-\gamma}{2}}$,

whence substituting,

$$\text{area } QOR = \frac{ab}{4} \tan\frac{\beta-\gamma}{2}.$$

$\therefore$ polar area $= S$

$$= \frac{16a^2b^2\,[PQR]^2}{a^3b^3 \tan\dfrac{\beta-\gamma}{2} \tan\dfrac{\gamma-\alpha}{2} \tan\dfrac{\alpha-\beta}{2}}$$

But $-b \tan\dfrac{\beta-\gamma}{2} \tan\dfrac{\gamma-\alpha}{2} \tan\dfrac{\alpha-\beta}{2}$
$=(ABC)=4(PQR)$;

$$\therefore \text{ polar area} = \frac{16a^2b^2(PQR)^2}{4a^2b^2(PQR)} = (ABC)$$

$=$area of original triangle.

13. Through any two points $A$ and $B$ on an equilateral hyperbola lines are drawn parallel respectively to the polars of $B$ and $A$: a circle may be described passing through the intersection of these lines, through $A$ and $B$, and through the centre of the hyperbola.

Let $A$, $B$ be points on the hyperbola, $xy=c$, centre $O$, then equations of sides of quadrilateral $OACB$ are

$$OA,\ \frac{x}{x_1} - \frac{y}{y_1} = O; \quad AC,\ \frac{x}{x_1} + \frac{y}{y_1} = C';$$

$$OB,\ \frac{x}{x_2} - \frac{y}{y_2} = O; \quad BC,\ \frac{x}{x_2} + \frac{y}{y_2} = C'';$$

whence $\alpha$, $\alpha'$ being angles $AO, B, ACB$

$$\tan\alpha = \frac{-\dfrac{1}{x_1y_2} + \dfrac{1}{x_2y_1}}{\dfrac{1}{x_1x_2} + \dfrac{1}{y_1y_2}};$$

$$\tan\alpha' = \frac{+\dfrac{1}{x_1y_2} - \dfrac{1}{x_2y_1}}{\dfrac{1}{x_1x_2} + \dfrac{1}{y_1y_2}};$$

$\therefore \tan\alpha + \tan\alpha' = O;\ \alpha + \alpha' = \pi.$

$\therefore$ Quadrilateral $OACB$ is inscriptible in a circle.

NOTE.—We are indebted to Professor Frisby for the solution of this problem.

14. The locus of the foot of the perpendicular drawn from the focus of a parabola on the normal is another parabola.

Let $y^2=4mx$ be $=$n of parabola. The length of perpendicular from $(m.o)$ on normal $2m(y-y')+y'(x-x')=0$ is

$$\rho = \frac{y'(x'+m)}{\sqrt{y'^2+4m^2}} = \sqrt{\{x'(x'+m)\}}.$$

Now, $\theta$ being angle made with the axis by perpendicular,

$$\sin\theta = \sqrt{\frac{m}{x'+m}}, \quad \cos\theta = \sqrt{\frac{x'}{x'+m}},$$

$$\therefore \rho = \frac{m\cos\theta}{\sin^2\theta}$$

$$\therefore \rho^2\sin^2\theta = m\rho\cos\theta$$

$$y^2 = mx = \text{n of locus.}$$

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RECENTLY in a book-store in the City of Mexico, a tourist from Yankeeland found a Spanish history of the United States, with the imprint of a Madrid publishing house. Its five hundred pages of miscellaneous reading matter furnished him with much very curious information. Lincoln's emancipation proclamation was made to relate to Indians instead of negroes. An incident of Indian bravery in King Philip's time was located in the War of the Rebellion. The characters in "Mrs Henriquetes Becker Stowe's" "Uncle Tom's Cabin" were given as historical. The pictures were as queer as the text. Lincoln was shewn with a cabinet party composed of Indian chiefs, New York was a small straggling village, and Washington had a monarch's crown on his head.