load element (see Fig. 3). The formulæ for δ_{mb} and δ_{mo} are analogous to that of δ_{ma}. The denominators of the equations for X_a, X_b and X_o are

$$\delta_{aa} = \int_{-\frac{1}{2}l}^{+\frac{1}{2}l} Z^2\,dx = l$$

$$\delta_{bb} = \int_{-\frac{1}{2}l}^{+\frac{1}{2}l} x^2\,dx = \frac{1}{12}l^3$$

$$\delta_{oo} = \int_{-\frac{1}{2}l}^{+\frac{1}{2}l} y^2\,dx + li^2 = \frac{4}{45}f^2l + li^2.$$

The equations for the statically indeterminate quantities and now known:

for $x < 0$, $$X_a = -\frac{1}{8}\left(1 + 2\frac{x}{l}\right)^2,$$

$$X_b = \frac{1}{2}\left(1 - \frac{x}{l}\right)\left(1 + 2\frac{x}{l}\right)^2.$$

For $x > 0$, $$X_a = -\frac{1}{8}l\left(1 - 2\frac{x}{l}\right)^2,$$

$$X_b = -\frac{1}{2}\left(1 + \frac{x}{l}\right)\left(1 - 2\frac{x}{l}\right)^2.$$

For both $x < 0$ and $x > 0$,

$$X_o = \frac{15}{64}\frac{l}{f}\,\alpha\left(1 - 2\frac{x}{l}\right)^2\left(1 + 2\frac{x}{l}\right)^2,$$

$$\text{where } \alpha = \frac{1}{1 + \dfrac{45}{4}\dfrac{i^2}{f^2}}.$$

The bending moments and normal forces in the arch can then be computed. The normal force N is made equal to X_o

$$N = X_o = \frac{15}{4}\frac{l}{f}\,\alpha\left[\left(\frac{x}{l}\right)^2 - \frac{1}{4}\right] = \frac{l}{f}N_1.$$

The value of N_1 depends solely on the value of the ratio $\frac{x}{l}$ and on the value of α. Below, N_1 is given for $\alpha = 1$ and $\alpha = .9$, and $\frac{x}{l}$ varying with differences of .1.

$\frac{x}{l}$ =	0	.1	.2	.3	.4	.5
$\alpha = 1$; N_1 =	.2344	.2160	.1654	.0960	.0304	.0000
$\alpha = .9$; N_1 =	.2110	.1944	.1489	.0864	.0274	.0000

The line of influence of the bending moment M at a point of the arch with the co-ordinates x and y has the equation

$$M = l\left(\frac{M_o}{l} - \frac{X_a}{l} - \frac{x_1}{l}X_b - \frac{y_1}{f}H_1\right) = l\,M_1;$$

M_o being the corresponding ordinate of the line of influence of the bending moment in the statically determinate auxiliary system. M_1 depends only on the ratios $\frac{x_1}{l}$ and $\frac{x}{l}$, where x is the abscissa of the line of influence. In the table on page 657 is given the values of M_1 for varying $\frac{x}{l}$ and $\frac{x_1}{l}$ and for $\alpha = 1$ and .9 respectively.

By the aid of these lines of influence the maximum and minimum bending moments are finally determined

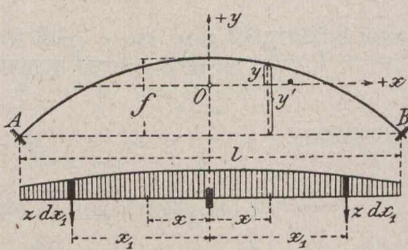

Fig. 3.

and also the simultaneously acting normal forces effected by a uniformly distributed live load p per unit length:

Maximum and Minimum Bending Moments.

	Max. M.	N.	Min. M.	N.
		$\alpha = 1$		
$\frac{x_1}{l}$ = —.5	+.0173 pl^2	.0853 $\frac{pl^2}{f}$	—.0173 pl^2	.0397 $\frac{pl^2}{f}$
—.4	+.0042 "	.0591 "	—.0042 "	.0659 "
—.3	+.0073 "	.0317 "	—.0073 "	.0933 "
—.2	+.0093 "	.0497 "	—.0093 "	.0753 "
—.1	+.0071 "	.0661 "	—.0071 "	.0589 "
0	+.0054 "	.0631 "	—.0054 "	.0619
		$\alpha = .9$		
$\frac{x_1}{l}$ = —.5	+.0118 pl^2	.0671 $\frac{pl^2}{f}$	—.0203 pl^2	.0454 $\frac{pl^2}{f}$
—.4	+.0026 "	.0355 "	—.0065 "	.0770 "
—.3	+.0072 "	.0283 "	—.0076 "	.0842 "
—.2	+.0103 "	.0476 "	—.0080 "	.0649 "
—.1	+.0089 "	.0661 "	—.0053 "	.0464 "
0	+.0075 "	.0686 "	—.0034 "	.0439

A uniformly distributed total load g per unit length gives

$$X_a = -\frac{gl^2}{24},\; X_b = 0,\; X_o = \frac{gl^2}{8f}\alpha,$$

and the bending moment at the various points of the arch is

$$Mx_1 = \frac{1}{24}gl^2\left(1 - 12\frac{x_1^2}{l^2}\right)(1 - \alpha).$$

For $\alpha = 1$, there are no bending moments and consequently the maximum and minimum moments produced by a uniformly distributed live load have the same numerical values.

An increase of the temperature of $t°$ gives

$$X_{at} = 0,\; X_{bt} = 0,\; X_{ct} = \frac{45}{4}EJ\,\alpha\frac{\epsilon t}{f^2}.$$