Mathematical Department.

FALLACIES.

1. Let $a=1$ and $x=1$ $\therefore a=x$. Multiply these equals by a and $a^2=ax$. Subtract x^2 from each side and $a^2-x^2=ax-x^2$. Factor both sides $(a+x)(a-x)=x(a-x)$. Divide both sides by $a-x$ and $a+x=x$. Substitute a for x since they are equal, and $2a=a$. Divide both sides by a and $2=1$.

2. Let $a=x$ $\therefore a-x=0$. Multiply both sides by 2 $\therefore 2(a-x)=0$. Hence we have $a-x=0=2(a-x)$. Divide this by $a-x$ and $1=2$, *thus confirming* the conclusions of (1).

3. The minute hand of a clock goes twelve times as fast as the hour hand goes. Suppose the clock shows the time to be XII o'clock precisely. The minute hand at the next beat of the pendulum begins to gain on the hour hand and must go round 60 minute divisions before it can get back to XII. Meantime the hour hand will have gone $\frac{1}{12}$ of 60 or 5 minute divisions and will be at I, so that the minute hand is 5 spaces behind the hour hand. Now while the minute hand is going over these 5 spaces, the hour hand will go over $\frac{1}{12}$ of 5 spaces or $\frac{5}{144}$ of a space, and the minute hand will be $\frac{5}{12}$ of a space behind. Then while the minute hand *goes over this* $\frac{5}{12}$ of a space the hour hand will go over $\frac{1}{12}$ of $\frac{5}{12}$ of a space or $\frac{5}{144}$ of a space, so that the minute hand will still be $\frac{5}{144}$ *of a minute* space behind the hour hand. Similarly while the minute hand goes over this $\frac{5}{144}$ space the hour hand will move ahead $\frac{5}{1728}$ of a minute space, and so on, $\frac{5}{20736}$, etc. On the whole it is clear that there must always be $\frac{5}{12}$ of a definite fraction of a minute space between the hands, consequently the minute hand can never exactly overtake the hour hand, and therefore, *a fortiori*, the minute hand of a clock can never pass the hour hand.

4. Take straight line A————B. Describe on it the square $ABCD$, making D opposite A and C opposite B, the square being above AB. Bisect AB in E. Join DE. Produce DE to meet CB produced downwards in F. Then in the triangles ADE and EBF the side AE=the side EB by construction; the angles DAE and EBF are equal, each being a right angle; the angles DEA and BEF are also equal, being vertically opposite. The triangle ADE is therefore=triangle EBF by I. 26.

Again from B draw BG perp. to EF. Then in the triangles DAE and EBG the angles at E are equal as before, and the angle at A is a right angle=angle BGE, and the side AE=side EB. Hence triangle DAE=triangle EGB by I. 26.

But by the former proof triangle DAE=triangle EBF. Hence triangle DAE and triangle EBG are each=triangle DAE and must themselves be equal, that is the whole triangle is=to the part of it. Hence the axiom ought to read "the whole is sometimes =to the part.

5. $\sqrt{-1}=(-1)^{\frac{1}{2}}$, by the theory of indices. But $(-1)^{\frac{1}{2}}=(-1)^{\frac{2}{4}}$ and this is=the fourth root of the second power of (-1), since the numerator always indicates a power and the denominator a root when we have a fractional exponent; *i.e.*, $(-1)^{\frac{2}{4}}=\{(-1)^2\}^{\frac{1}{4}}$. But $(-1)^2=+1$ $\therefore \{(-1)^2\}^{\frac{1}{4}}=(+1)^{\frac{1}{4}}$. Now the fourth root of $+1$ is $+1$, for $(+1)(+1)(+1)(+1)=+1$. Hence on the whole $\sqrt{-1}=+1$, and therefore the square root of -1 is not impossible. Also $\sqrt{+1}=+1$, $\therefore \sqrt{-1}=\sqrt{+1}$ and $\therefore -1=+1$.

6. "The study of mathematics educates to no sagacity in detecting and avoiding the fallacies which originate in the thought *itself of the reasoner.*" "Mathematical reasoning allows no room for any sophistry of thought." "A mathematician is not compelled to be on his guard against the fallacies which beset the route of the ordinary reasoner." "A man is made to reason justly in mathematics in the same manner in which a man is made to walk straight in a ditch." "It requires a most ingenious stupidity to go wrong" in "a science in which there is no reasoning wrong." —Sir Wm. Hamilton, *in various works.*

If Sir Wm. Hamilton is correct—*which we do not assert*—we hope the readers of this Department will not allow themselves to lie under the charge of "a most ingenious stupidity." He says, "Mathematics are the easiest of all sciences; their perspicuity is excessive. A mathematical reasoning may certainly transgress in form, and a railway locomotive may go off the rails. To minds of any talent mathematics are only difficult because they are too easy." Therefore we invite our readers who have "minds of any talent" to point out in the above, especially in No. 3 and No. 4, just where the locomotive of thought was derailed. The conclusions are certainly false, at what point does the reasoning "transgress in form"? In case the perspicuity does not prove very excessive, it may be well to apply to Sir Wm. himself for light—say to his *Logic*. *Locke* and *Reid* give no countenance to his assertions, but we have no more space for quotations.

7. The following false solution appeared a year or two ago in the *British Mechanic:*

Given $397\frac{1}{4}x^{760}=400x^{759}-2\cdot75$ to find x.

Solution. $1589x^{760}=1600x^{759}-11$,

$\therefore 1600x^{759}(x-1)=11(x-1)$. $\therefore x=1$, one root.

$$1600x^{759}=11, \text{ and } x=\left(\frac{11}{1600}\right)^{\frac{1}{759}}.$$

Note.—Perhaps it is only fair to add some further examples of fallacies at our own expense. We give our readers our hearty consent to detect and correct those rather prodigious ones on pages 103 and 104 of the May No. We feel sure all our friends would enjoy the holidays better if they could see the avalanche of correspondence that has come down upon the unprotected editor *in re* aforesaid fallacies. If it were a question of politics or theology, we might escape but in the imperial domain of mathematics sophistry will not pass muster. We have, however, appreciated the kindness and courtesy of the numerous friends who took the trouble to point out the mistakes and we return them cordial thanks. The following are some of the slips which occurred:

No. 1, part 2. The second 7 should be $\frac{7}{4}$ and the result $3\frac{47}{56}$.

No. 2. The correct result is $43\frac{1}{3}$ seconds, *i.e.*, L.C.M. of $1\frac{1}{12}$ and $\frac{10}{9}$ seconds.

No. 4. "C in 100 hrs." should read C in $\frac{6}{5}$ with corresponding corrections throughout.

No. 4. 59 days is correct, since the *true* discount is required. But $\frac{59}{365}$ of 8 should evidently read $\frac{59}{365}$ of $\frac{8}{100}=\frac{472}{36500}$ instead of $\frac{472}{365}$, and the discount is $\frac{472}{36573}$ or $\frac{118}{9143}$ of the face, $\therefore$ P.W. $=\$493.616$.

No. 5. 75 lbs should read 70 lbs, and advance=40 % not 50 %.

No. 8. The solution is correct, but the problem is misprinted. It should have read, "the cost per mile is equal to as many pence as there are miles."

We wish all our readers a joyous holiday free from all fallacious hopes, and we trust that these amusing mistakes may help to cure them of dyspepsia in case they raise *π* too frequently.

Correspondence.

To the Editor of the Canada School Journal.

Will you please answer the following questions in your next issue.

1. Does the History for the next Intermediate Examination include English, Canadian & Roman History?

2. What are the subjects for the non-professional examination for First Class C in 1885?

3. Is General History required?

4. Can a teacher who has taught for several years on a 2nd class certificate, if successful in passing the non-professional examination for 1st C present herself for examination at the professional examination without previously attending a session at the Normal School?

Yours Truly,

Minnie Smith.

Flesherton May 26th 1884.

Reply—1. Yes. 2. See Canada School Journal page 75, April No. The other subjects are fixed, apply to the Department for course of study. 4. Ask the Deparment; we believe the answer is, Yes, but you had better get an offical reply.

To the Editor of the Canada School Journal.

Dear Sir.—It is with pleasure that I have read your articles on "Canadian History" and "The Eleventh Plague" in the May number of the Journal." Please permit a word or two, I believe that the custom officials of Windsor have confiscated several of the foul publications you mention, as being obscene literature. May not the allegations that you have made be equally applicable to many of the serials of our weekly and daily newspapers. Take M. E.