# The Canadian Engineer 

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# Design of Turbine Runner Bands 

With Special Reference to High Speed Runners - Detailed Analysis of Hydraulic Conditions at the Band-Development of Simple Formula to Determine Angle of Band with Vertical

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THAT a detailed analysis should be made of hydraulic conditions at the band of high speed runners, was suggested some time ago to the writer by Frank B. Fish, chief engineer of James Leffell \& Co. Mr. Fish believed that the band angle with the vertical should be increased quite a little; he contended further that the band when so inclined would discharge more water by reason of its outward component. This angle, to eliminate pressure against the band, is coincident with the resultant of the horizontally acting centrifugal forces and the vertical pressure forces.

Referring to Fig. I, and considering a slice of area $A$ and thickness $d l$, we have the following forces acting: The velocity head, the pressure head, acceleration due to gravity, the pressure below the slice, centrifugal force, friction head, and suction head.

In the following formulæ, $A$ is the area in square feet; $\gamma$, the specific weight of water per cubic foot; $v$, true velocity of water in space in feet per second; $H_{p}$, head in feet untransformed into velocity; $g$, acceleration due to gravity; $r^{\prime}$ and $r^{\prime \prime}$, radii in feet to the point in question; $w$, angular velocity in radians per second; $e$, the mechanical efficiency.

In this investigation we will consider forces rather than velocities in order to facilitate mathematical transformations.

Due to velocity, we have a force

$$
\begin{equation*}
F_{v}=A \gamma\left(v^{2} / 2 g\right) \cos Q \tag{I}
\end{equation*}
$$

acting yertically.
Due to pressure head,
$F_{\mathrm{p}}=A \gamma H_{\mathrm{p}}$
Due to gravity,
$F_{\mathrm{w}}=(A d l \gamma / g) g=A d l \gamma$
Pressure from below,
$F_{\mathrm{b}}=-A d h \gamma$ acting upwards.

## Centrifugal force,

$F_{0}=(A d l \gamma / g) r w \omega^{2}$
acting horizontally.
Due to friction, which may be conveniently expressed as part of the total or static head, $H_{\mathrm{s}}$,

$$
\begin{equation*}
F_{t}=-A \gamma H_{\mathrm{s}}(\mathrm{I}-e) \tag{VI}
\end{equation*}
$$

Due to suction: Due to the centrifugal force generated by the runner, especially at the band, the water is drawn
into the runer with an artificial pressure which may be expressed as a force of
$F_{\mathrm{s}}=\left[\left(\mathrm{r}^{\prime \prime 2} w^{2} / 2 g\right)-\left(\mathrm{r}^{1^{2}} w^{2} / 2 g\right)\right] A \gamma \ldots \ldots \ldots \ldots \ldots . .$.


Fig. No. 1
The total horizontal force is that due to centrifugal: force

$$
\begin{equation*}
\Sigma_{\mathrm{h}}=F_{\mathrm{c}}=(A d l \gamma / g) r^{\prime} \gamma w^{2} \tag{VIII}
\end{equation*}
$$

and the total vertical force equals

$$
\begin{equation*}
\Sigma_{\mathrm{w}}=\mathrm{F}_{\mathrm{v}}+\mathrm{F}_{\mathrm{p}}+\mathrm{F}_{\mathrm{w}}-\mathrm{F}_{\mathrm{b}}-\mathrm{F}_{\mathrm{f}}+\mathrm{F}_{\mathrm{s}} \tag{IX}
\end{equation*}
$$

The tangent of the angle then, that the resultant of the horizontal and vertical forces makes with the vertical

