$y = a' + \frac{1}{z}$; $z = a'' + \frac{1}{w}$, &c.

Hence $x = a + \frac{1}{a'} + \frac{1}{a''} + \frac{1}{w}$,

or approximately, omitting the w,

$$x = \frac{a''(aa'+1)+a}{a''a'+1}.$$

By repeating the operations we may approximate with any degree of closeness to the value x, though the numerical labor of the operatives will continually increase.

Example: $x^3+10x^2+6x-120=0$.

When $x=2$, the left hand side becomes negative, and when $x=3$. positive. A root lies between 2 and 3. Let $x = 2 + \frac{1}{y}$, and substitute. Then

$$1+16y+58y^2-60y^3 = 0.$$

$y=1$, $y = 2$ give contrary signs. Let $y = 1 + \frac{1}{z}$, and substitute. Then

$$-60-122z-48z^2+16z^3 = 0,$$

$z = 4$, $z = 5$ give contrary signs. Let $z = 4 + \frac{1}{w}$, and substitute. Then

$$16+144w+262w^2-292w^3 = 0.$$

and here $w = 1$, $w = 2$ give contrary signs.

Hence, approximately, $x = 2 + \cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1}}} = 2.833.$

To be certain that the third figure is correct we should have carried the operations one stage further, and if we still got for the first three decimals 833, we have sufficient evidence that the third figure is 3.

We are sure many of our readers, in applying their algebra to geometrical problems, have frequently encountered cubic and biquadratic equations and been unable to proceed. The above will often extricate them from such difficulties.

APPROXIMATE QUADRATURE OF THE CIRCLE.

The following mechanical quadrature (approximate) of the circle is given by P. E. Chase, LL.D., in the Proceedings of the American Philosophical Society:

Let AC ($=20$, say) be the diameter of the circle a straight line approximately equal to whose circumference it is required to find. From AC cut off $AB=3$. Draw AD at right angles to AC and $=9$. Join CD, and draw BE parallel to CD, meeting AD in E. Produce AC to X, making $AX=60$. Produce EA to Y, making $EY = 20$. Then XY shall be approximately equal to the circumference of the circle whose diameter is AC. By calculation it may be easily shown that $XY = 3{\cdot}141585\ AC$, which is certainly sufficiently accurate for practical purposes. The method, requiring only a divided rule, a square, and parallel rulers, will be found useful in many mechanical operations.

SOLUTIONS OF PROBLEMS IN JANUARY NUMBER.

1. If O be the centre of the ball 18 inches in diameter, a point 9 inches above O will be the highest point of this ball above the floor. Let A, B, C be the centres of the other balls whose diameters are respectively 16, 20 and 28. Then the sides of the tetrahedron $OABC$ are known. Frost (Solid Geometry, §124) gives a relation between the four-point co-ordinates of a plane. Three of the co-ordinates are known, viz.: the perpendiculars from A, B, C on the floor, and from this relation the fourth co-ordinate, the perpendicular from O, may be found.

2. Let x, y be the co-ordinates of the officer at time t from starting, s the distance travelled by him in this time, the centre of the army being supposed to move along the axis of x. Then $s = 7t$. Also, 1 being the radius of the circle

$$\{x-(3t+1)\}^2+y^2=1^2,$$

$$\therefore\ x-(\tfrac{3}{7}s+1) = \pm\sqrt{1-y^2},$$

$$\text{or } s = \tfrac{7}{3}[x-1\mp\sqrt{1-y^2}] \ldots\ldots(1).$$

$$\text{Also } s = \int\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx \ldots\ldots(2).$$

Eliminating s from (1) and (2), we would then have the equation to the curve; and then, knowing x in terms of y, we could from (1) find s when $y=0$.

3. Let $u\left(=\frac{1}{r}\right)$, θ be the polar co-ordinates of the dog at time t, the centre of the circle being the pole, and a diameter through the point from which the dog starts being the initial line. The equation to the tangent to the curve in which the dog moves is

$$u' = u\cos(\theta'-\theta)+\frac{du}{d\theta}\sin(\theta'-\theta), \ldots\ldots(1).$$

And the dog's motion being always directed towards the rabbit, this tangent will always pass through the point $\left(a, \frac{50kt}{a} + \alpha,\right)$ where $a =$ radius of field, $50k =$ rate of motion of rabbit, and $\alpha = 85°$. Hence, substituting in (1),

$$a = u\cos\left(\frac{50kt}{a}+\alpha-\theta\right)+\frac{du}{d\theta}\sin\left(\frac{50kt}{a}+\alpha-\theta\right) \ldots(2).$$

Also,

$$51kt = s = \int\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}.d\theta = \int\frac{1}{u}\sqrt{u^2+\left(\frac{du}{d\theta}\right)^2}.d\theta \ldots (3)$$

From (2) and (3) t is to be eliminated; we would then have the equation to the curve in which the dog moves, and could then from (3) find s when $u = \frac{1}{a}$.

4. The solution of this may be effected by using the geometrical construction given in the appendix to Todhunter's Euclid, p. 305, *et ante*.

5. Let V be the quantity of water in the tub at time t, dV' the quantity of water that comes from the spout in time dt, of which let dV be the quantity that remains in the tub. On mixing there will be

$$\frac{40}{40+dV'}(V+dV')$$

gallons of water in the tub. Hence quantity of water added

$$= \frac{40}{40+dV'}(V+dV')-V$$

$$= \frac{(40-V)dV'}{40+dV'}$$

Hence

$$\frac{(40-V)dV'}{40+dV'} \text{ or } \frac{(40-V)dV'}{40} = dV.$$

$$\therefore \frac{dV}{40-V} = \frac{1}{40}dV' = \frac{3}{40}dt.$$

Integrating,

$$-\log(40-V) = \frac{3}{40}t + C;$$

$$\therefore -\log 40 \quad = 0 + C:$$

$$\therefore \log\frac{40}{40-V} = \frac{3}{40}t$$

$$\text{and } V = 40\left(1-e^{-\frac{3}{40}t}\right);$$