a, c, e, h, θ and ϕ separately. Thus a solution of the given quintic is effected. It will be pointed out how a, c, e, h, θ and ϕ can be found separately, should we desire to obtain their values.

THE PROOF.

Case in which p_3 is zero.

§4. When $p_3 = 0$, the investigation is much simplified. By beginning with this case, and presenting a full description of it, we shall be prepared for giving an exposition, less detailed, but still sufficiently minute to make the theory intelligible, of the case in which p_3 is not assumed to be zero. When p_3 is zero, equations (2) and (5) become

$$\left.\begin{aligned} u_1u_4 &= a\sqrt{z} \\ u_2u_3 &= -a\sqrt{z} \\ \text{and} \qquad A &= -\frac{he(\theta^2-\phi^2 z)}{a} \end{aligned}\right\} \tag{7}$$

Since B is the coefficient of the rational part, B' the coefficient of $\sqrt{z}$, B'' the coefficient of $\sqrt{(hz+h\sqrt{z})}$, and B''' the coefficient of $\sqrt{z}\sqrt{(hz+h\sqrt{z})}$, in the expansion of u_1^5, their values are given by the equations

$$\left.\begin{aligned} a^2zB &= 2k(k^2-c^2z) - a^3cz^2 + 2czhe(\theta^2-\phi^2z) \\ a^2zB' &= 2c(k^2-c^2z) + a^3kz + 2khe(\theta^2-\phi^2z) \\ a^2zB'' &= 2(k^2-c^2z)\theta - \frac{2(k^2+c^2z)(\theta+\phi z)}{c} + \frac{z(\theta+\phi)(4kc+a^3z)}{e} \\ a^2zB''' &= 2(k^2-c^2z)\phi + \frac{2(k^2+c^2z)(\theta+\phi)}{e} - \frac{(\theta+\phi z)(4kc+a^3z)}{e} \end{aligned}\right\} \tag{8}$$

The equations (6), when p_3 is zero, become

$$\left.\begin{aligned} p_4 &= -20A + 15a^2z \\ p_5 &= -4B + 40acz \\ B'' &= 1 \\ B''' &= 0 \\ hz(\theta^2+\phi^2z+2\theta\phi) &= k^2+c^2z \\ h(\theta^2+\phi^2z+2\theta\phi z) &= 2kc+a^3z \end{aligned}\right\} \tag{9}$$

Let

$$\left.\begin{aligned} y &= a^2z \\ \text{and} \qquad t &= \frac{c}{a} \end{aligned}\right\} \tag{10}$$