PROPOSITION 10.

THEOREM.

FIG 9. From the point D′, with the distances D′e″ and Df″, and from the point B, with the distances Bk and Bi, describe the intersections a and b, and through the points a and b, draw the straight line abn, meeting the circular arc BD′ in the point n; then from C as a center with the distance Cn, describe through n the arc n′nN′, and on the arc BD make the arc Bn equal to the arc Bn′—and join A and n.—The distance An is equal to the circumference of the circle ABD, (Fig. 4.)

For it is demonstrated (prop. 1.) that the point n, is the point of ultimate intersection of the line ab &c., or of all the intersections described through the series of points of variation e″ and f″, &c., and through the points k and i &c—through which the curve line ab ... n shall coincide with its chord an (Lem. 3.); therefore the point n must be in the arc n′N′ (Fig. 5.); but Bn is equal to Bn′ (Fig. 5.)—hence An (Fig. 9.), is equal to the circumference of the circle ABD (Fig. 4.)

SOLUTION EIGHTH.

PROPOSITION 11.

THEOREM.

FIG. 9. From the point A as a center with the distances Aq and Ao′ describe the arcs qq′ and o″x, and from B as a center with the distances Bq′ and Bx′ describe the arcs q′q″ and x′x″—also from C as a center with the distances CK and Co, describe the arcs Ky′ and op′—and from B as a center with the distances By′ and Bp′, describe the arcs y′y″ and p′p″—and with the distances Bk′ and Bi′, describe the arcs k′k″ and i′i″—then from B as a center with the distances Bk″ and Bi″, and from D′ as a center with the distances De and Df, describe the intersections a′ and b′, and through the points a′ and b′ draw the straight line a′b′n meeting the circular arc BD′ in the point n; then from C as a center through the point n, describe the arc n′nN′, and on the arc BD, make the arc Bn equal to the arc Bn′, and join A and n.—The distance An is equal to the circumference of the circle ABD (Fig. 4.)

The demonstration of this evidently must be the same as prop. 10.

SOLUTION NINTH.

PROPOSITION 12.

THEOREM.

FIG. 9. Let the straight line abn be drawn through the points of intersections a and b, and the straight line a′b′n drawn through the points a′ and b′, intersecting each other in the point n—also from the point C as a center, through n describe the arc n′nN′, meeting the arc BC in the point n′, and the arc CD in the point N′,—and on the arc BD make the arc Bn equal to the arc Bn′, and join A and n.—The distance An shall be equal to the determinate length of the circumference of the Circle ABD, Fig. 4.

For the point n is the ultimate intersection of the intersections of each of the lines ab ... n and a′b′ ... n, upon the circular arc Bdef ... hikD′ (prop. 2 and 3), and the points d,e,f ... h,i,k, are the intersections of the infinite series of arcs 1″T′, 2″S′, and 3″R′, &c., and of the infinite series of arcs 2′K′, 3′O′ and 4′Q′, &c.—hence the point n must be upon the arc n′nN′, Fig. 5, but Bn′ is by construction equal to Bn, Fig. 5—hence Bn, Fig. 9, must be equal to Bn Fig. 5, and An Fig. 9 is equal to the circumference of the Circle ABD, Fig. 4.

FINIS.