48. In the figure of (47), if F be the point of intersection of BE with AC, and FG be drawn to CD, parallel to BC, and DH perpendicular to BE, shew that the angle DFG is equal to DAG.

49. In the figure of (48) shew that the rectangle HE, EF is equal to a twelfth of the whole square.

(47, 48, and 49 are from the Matriculation of 1866).

50. A quadrilateral figure possesses the following property: Any point being taken, and four triangles formed by joining this point with the angular points of the figure, the centres of gravity of these triangles lie on the circumference of a circle; prove that the diagonals of the quadrilateral are at right angles to each other.

51. Through any point of a chord of a circle other chords are drawn; shew that lines from the middle point of the first chord to their middle points will meet them all at the same angle.

52. Prove analytically (1) The angle in a semi-circle is a right-angle; (2) Angles in the same segment of a circle are equal.

53. An ellipse and hyperbola that have the same foci and centre will cut one another at right angles.

54. In No. (53), if from any point in the circumference of the circle which passes through the points of intersection of the ellipse and hyperbola, tangents be drawn to those curves, they will be at right angles.

55. Find the condition in order that two given equations of the second order may represent similar and similarly situated curves.

56. Solve
$$\left.\begin{aligned} x + y + z &= 11 \\ x^2 + y^2 + z^2 &= 49 \\ yz &= 3x\,(z-y) \end{aligned}\right\}$$

57. If there be n straight lines lying in one plane the No. of different n-sided polygons formed by them is $\frac{1}{2}\,\underline{|n-1}$

58. Resolve $2x^2 - 21xy - 11y^2 - x + 34y + 3$ into factors of the first degree.

59. Find the continued product of n such trinomials as $x^2 - ax + a^2$, $x^4 - a^2x^2 + a^4$, $x^8 - a^4x^4 + a^8$ etc.

60. Prove (by the method of Indeterminate Co-efficients) that the sum of the products of the first n natural numbers, taken two and two together, is
$$\frac{n\,(n-1)\,(n+1)\,(3n+2)}{24}$$

61. Show that the remainder after n terms of the expansion of $(1-x)^{-2}$ is
$$\frac{(n+1)\,x^n - nx^{n+1}}{(1-x)^2}$$

62. The sum of the first $\overline{r+1}$ co-efficients of the expansion of $(1-x)^{-m}$ is $\dfrac{\underline{|m+r}}{\underline{|m}\ \underline{|r}}$ m being a +ve integer.

63. A body is floating between two known fluids, and the part immersed in the lower is observed to be the same as if it were floating on the surface of a fluid formed by the mixture of equal portions of the two fluids; determine the specific gravity of the solid.

64. Two hollow cones, filled with water, are connected together by a string attached to their vertices, which passes over a fixed pulley; prove that, during the motion, if the weight of the cones be neglected, the total pressures on their bases will always be equal, whatever be the forms and dimensions of the cones. If the height of the cones be h, h^1, and heights mh, nh, be unoccupied by water, the total normal pressures on the faces during the motion will be in the ratio:
$$n^2+n+1 \;:\; m^2+m+1$$

65. A hollow cone floats with its vertex downwards in a cylindrical vessel containing water. Determine the equal quantities of water that must be poured into the cone and cylinder that the position of the cone in space may not be altered.

66. At what angle must two mirrors be inclined so that a ray incident parallel to one of them, may, after reflection at each be parallel to the other?

67. Three circles are so inscribed in a triangle that each touches the other two and two sides of the triangle; prove that the