13, Solve the following equation without completing the square : $\frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{x}{y} + \frac{y}{x} = 6\frac{3}{4}$; and $x - y = 2$.

14. The sum of four numbers in geometrical progression is 45, and sum of their squares 765 ; what are they ?

15. A hollow sphere, whose inner diameter is 3 feet, is filled with water ; what is the ratio between the pressure on the internal surface of the sphere and the weight of the water ?

I am, Sir,

Your obedient Servant,

A. DOYLE.

Hamilton, July, 1872.

PUBLIC SCHOOL EXAMINATIONS.

To the Editor of the Journal of Education.

SIR,—I send you, for publication in the *Journal*, solutions of the questions in Natural Philosophy and Algebra proposed to candidates for first-class certificates at the recent examination of Public School Teachers. In your next number I may perhaps make some remarks of a general kind on the result of the examination.

I have the honour to be, Sir,

Your obedient servant,

GEORGE PAXTON YOUNG.

TORONTO, 1st August, 1872.

NATURAL PHILOSOPHY.

1. A considerable number of students have still very vague ideas of what a uniformly accelerating force is, and how it is measured. I therefore crave attention to the following statements by the late Dr. Whewell, of Cambridge : "The magnitude of forces is measured by their effects ; and the effect of forces which we consider in Dynamics is velocity. Accelerating force is force measured by the velocity which, in a given time, it would add to the motion of a body. If the velocity added be equal in equal times, the force is said to be uniform."

Let x be the time which the particle P takes to reach A. In that time it goes over a space $20x$ in virtue of the velocity already acquired ; and over an additional space of $16\,x^2$ due to the accelerating force to which it is subject. Therefore

$$16x^2 + 20x = 6 \therefore x = \tfrac{1}{4}.$$

Similarly, if y be the time in which Q reaches A, we have

$$20y^2 + 20y = 6\tfrac{1}{4} \therefore y = \tfrac{1}{4}.$$

Therefore the particles reach A in the same time.

2. The ascending particle has, at A, a velocity of 8 feet a second. To destroy this velocity $\frac{1}{4}$ of a second is necessary. Another $\frac{1}{4}$ of a second is expended in the return of the particle from rest to A. Therefore the descending particle takes $\frac{1}{2}$ a second to reach the ground from A. In that time it goes through 4 feet in virtue of the velocity already acquired ; and 4 feet besides, due to the action of the force of gravity. Therefore

$$m = 8.$$

3. As this question has been satisfactorily treated by very few of the candidates, I give two solutions.

The forces represented by P A and P B have, as their resultant, a force acting in the direction of the diagonal of the parallelogram of which A P and B P are adjacent sides. But these forces, by supposition. keep the lever at rest. Therefore their resultant must pass through the fulcrum ; for if it struck the lever on either side of the fulcrum, it would turn the lever. Hence C is the point of intersection of the diagonals of a parallelogram, and therefore A C = B C.

Another solution : Draw C D perpendicular to A P, and C E to B P. Then, since the lever is at rest, the force at A multiplied by C D is equal to the force at B multiplied by C E. That is,

$$\text{P A} \times \text{C D} = \text{P B} \times \text{C E}.$$

Therefore, triangle A C P = triangle B C P.

$$\therefore \text{A C} = \text{B C}.$$

4. This question, with the preceding, appears to have been felt to be more difficult than any others in the paper. This shows, I think, that the candidates generally have no firm grasp of the principles of the resolution of forces. I will therefore give the solution of the question somewhat fully.

A force represented in magnitude and direction by A D can be resolved into two others ; one in the direction A B, and represented in magnitude by A B ; the other in a direction perpendicular to A B, and represented in magnitude by B D. This is a direct consequence of the principle of the parallelogram of forces, as may be seen by completing the parallelogram A B D E, and observing that the force represented in magnitude and direction by A D is a resultant of the forces represented in magnitude and direction by A B and A E respectively. Therefore a force $\sqrt{2}$ in direction D A, has for its resolved part in direction B A a force less than $\sqrt{2}$ in the proportion of B A to A D, or $\frac{\text{B A}\sqrt{2}}{\text{A D}}$ that is one.

In like manner, if the given force of 2 feet in the direction A C be resolved in the direction A B, and in that at right angles to A B, we shall find the former resolved part to be 1. But these forces, uniting in the direction B A, and uniting in the direction A B, counterbalance one another, leaving only forces the direction of whose action is at right angles to A B.

5. No note on this question seems necessary.

6. The pressure of 1 lb. sinks the cube one-sixth part. Therefore one-sixth part of a cube of water, of the same size as the given cube, weighs 1 lb., and the whole of such a cube of water weighs 6 lbs. But the given cube has only one-third of the specific gravity of water. Therefore its weight is 2 lbs.

The content of the sphere is $\frac{32\,\pi}{3}$; that of the cylinder is $\frac{28\pi}{3}$

Let h be the height of the barometric column. Then

$$28 : 32 = 7 : 8 = h + t : h + 5\tfrac{3}{7}$$
$$\therefore h = 30.$$

8. Bookwork.

9. The volume of the instrument is V ; that of the part not immersed in the first fluid kd ; therefore that of the part immersed is $V - kd$. Hence the weight of the fluid displaced is $S_1.(V - kd_1)$. But this represents the weight of the instrument. In like manner $S_2\,(V - kd_2)$ represents the weight of the instrument Therefore the quantities

$S_1\,(V - kd_1)$ and $S_2\,(V - kd_2)$ are equal to one another, and

$$\frac{S_2}{S_1} = \frac{V - kd_1}{V - kd_2}$$

[In the examination paper, the expression $\frac{S_2}{S_1}$ was, by an error of the press made $\frac{S_1}{S_2}$.]

10. Here f, in the formula, $s = \frac{1}{2}ft^2$, is less than 32 in the proportion of 1 to 11. Therefore $8 = \frac{16t^2}{11}$, and $t = \sqrt{5\cdot5}$.

ALGEBRA.

1. Let x be the number of minute spaces gone over by the minute hand since 3 o'clock. Then $x - 30$ is the number gone over by the hour hand. Therefore

$$12\,x - 360 = x, \text{ and } x = 32\tfrac{8}{11}.$$

2. Let m and n be the quantities. Then

$$x = \frac{m + n}{2}$$

$$y = \frac{2\,mn}{m + n}.$$

Therefore $xy = mn$, the geometrical mean between m^2 and n^2.

3. Let y and z be the roots. Then

$$y^3 + z^3 = 19, \text{ and } y + z = 1.$$
$$\therefore y = 3,\ z = -2.$$

By the substitution of either of these values in the given equation, we get $p = 6$.

4. The square root of $22 - 12\sqrt{2}$ found by the ordinary method is $2 - 3\sqrt{2}$. Therefore

$$10\,x + 2\sqrt{2} = (2 - 3\sqrt{\ }\ )\,(5\,x - 2\sqrt{2}\ .)$$
$$\therefore 5\,x = 2\sqrt{2} - 2.$$

5. Add 2 to each side. Then

$$\left\{x + \frac{1}{x}\right\}^2 + \left\{x + \frac{1}{x}\right\} = \frac{35}{4}.$$