

Sol—The fifth term of $(2^3-2)^{-n}$ is

$$\frac{n(n+1)(n+2)(n+3)}{1 \cdot 2 \cdot 3 \cdot 4} 2^{-3n-12} 2^4$$

$$= \frac{n(n+1)(n+2)(n+3)}{1 \cdot 2 \cdot 3 \cdot 4} 2^{-3n-8}$$

$$\sqrt[3]{\frac{1}{8}} = 6^{-\frac{1}{3}} = (2^3-2)^{-\frac{1}{3}}$$

$$= \frac{1}{2} + \frac{1}{3 \cdot 2^3} + \frac{1 \cdot 4}{1 \cdot 2 \cdot 3^2 \cdot 2^5} + \dots$$

10. Sum the series

$$(1) \frac{1}{\sqrt{2}} - \frac{1}{3} + \frac{\sqrt{2}}{9} - \frac{2}{27} + \dots \text{ to infinity.}$$

$$(2) 3+6+11+20+37 \text{ \&c., to } n \text{ terms.}$$

Sol.—In (1) the com. ratio is $-\frac{\sqrt{2}}{3}$ $\therefore$ the sum to infinity is

$$\frac{1}{1 + \frac{\sqrt{2}}{3}} = \frac{3}{\sqrt{2}(3 + \sqrt{2})}$$

The second series may be arranged thus—
 $(1+2)+(2+2^2)+(3+2^3)+(4+2^4)+\&c.$, thus it resolves itself into the two series

$$1+2+3+4+\&c., \text{ and } 2+2^2+2^3+2^4+\&c.,$$

$$\therefore \text{ sum is } \frac{n}{2} (1+n) + 2(2^n-1).$$

SOLUTIONS TO THE EXERCISES IN TODHUNTER'S EUCLID.

309. ABCD the quad., AD, BC meeting in P and AB, DC in Q; join RP, RQ, RC; then ang. CRQ, CBQ are tog. eq. two rt. ang.; also CRP, CDP; but CDP, CBQ are tog. eq. two rt. ang; hence CRP, CRQ eq. two rt. ang.

310. Describe a cir. about ABC; then evidently each of these bisecting lines also bisects the arc AB; hence ADBC is a quad. in a cir.

311. Draw DH tang. to cir. CDE cutting AB in H; then ang. HDE eq. DCE in alt. seg. eq. DAH; hence H is cen. of ABD.

312. The lines bisecting OA, OB, OC, OD at rt. ang. meet in the cens. of these cir. and evidently form a parallelogram.

313. Ang. DCE (eq. ang. DEC) eq. ang. EAC and ECA; but DCB eq. EAC (in alt. seg.); hence BCE eq. ACE.

314. F is the cen. of the cir. circumscribing ABC; and since the base BC and the vert. ang. BAC are const., therefore the cir. is const.; hence AF, its rad., is const.

315. Take O the middle pt. of BC; then the sq. on AB eq. sqs. on AD, DB tog. with twice the rect. BD, DO eq. sq. on AD, and rect. BD, DC eq. sq. on AD, and rect. AD, DE eq. rect. EA, AD; hence, &c.

316. E the pt. of contact, then the tang. at E makes with EC an ang. eq. to EAB, and with ED an ang. eq. to EBD; and their differences are eq.; hence AEB eq. DEC.

317. Describe a cir. passing through A, B and touching the given cir. at P, (Todhunter's Euclid, p. 299), P shall be the pt. reqd.

318. A the pt. where the lines meet. B the cen. of the given cir.; join AB cutting the cir. in C; in the cir. take D and E on opp. sides of AB, such that CD, CE each eq. half the reqd. seg.; apply No. 6, p. 296, (Todhunter's Euc.) to des. a cir. passing through D, E and touching one of the lines; this will evidently touch the other line and be the cir. reqd.

319. Let AD, BE, CF meet in G; then a cir. may be desc'd. about AFGD; hence the