constants involved are made equal to unity. It will be convenient therefore, to consider the case of least annual cost. The problem may be stated as follows:—

Given a pipe of fixed length, l, and under a given head, h, with a given discharge, q, it is required to find the size of the pipe so that for a given or assumed total loss of head, h', in the pipe the annual cost of the pipe will be a minimum.

The problem as thus stated is of considerable interest. In the "Engineering Record" of December 20th, 1913, page 682, E. R. Bowen describes the inverted Jawbone Syphon of the Los Angeles aqueduct. This inverted syphon was to be designed to contain the least amount of metal subject to the condition that the loss of head due to friction in the syphon should be 26 feet. The solution as given by Mr. Bowen is a graphical one requiring a great deal of labor. This same problem could have been solved in a much simpler way.

The annual cost of the pipe is the interest and depreciation on first cost and this will be represented by I. The The problem then is to find the size of the pipe so that I will be minimum, subject to the condition that the total loss of head in the pipe shall equal h'.

In practice, the pipe is made to consist of two or more sections, each section of constant diameter and thickness of shell. The greater number of sections, the more nearly will this pipe approach the theoretically most economical pipe, one in which the diameter decreases as the head increases.

For the sake of convenience we will assume that the pipe is to consist of three sections as shown in Figure I.

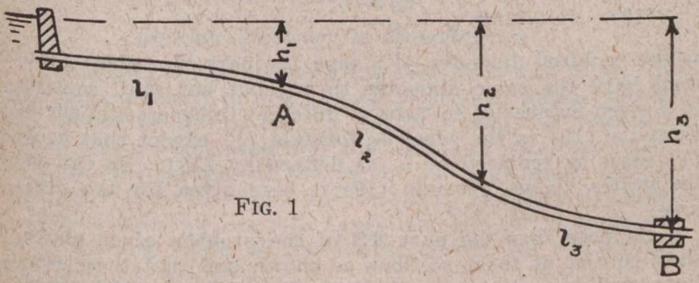

FIG. 1

Let l_1, h_1, d_1; l_2, h_2, d_2 and l_3, h_3, d_3 be the lengths, heads and diameters of the respective sections.

Evidently, the location of the points of division a and b will effect the economy. It can be shown that the location of the most economical points of division depends upon the profile. These points, perhaps, can best be found by trial. For our purpose it is necessary to assume that the points of division are given. Consequently h_1 and l_1, h_2 and l_2, h_3 and l_3 are assumed as given or determinate.

The interest and depreciation on first cost of a section is proportional to the amount of metal in that section. Using a well known principle of hydraulics, the amount of metal in that section under high head is found to be proportional to $d_r^2 h_r l_r$. The subscript r is used to indicate that it applies to a section and not to the pipe as a whole. The annual cost (interest and depreciation) of this section may therefore be represented by the equation

(7) $I_r = A d_r^2 h_r l_r$;

where A is a constant. Moreover, the friction head for this section may be expressed by (see eq. 3)

$$h'_r = \frac{f l_r}{d_r} \cdot \frac{v_r^2}{2g}$$

or, if v is eliminated by means of the relation $F_r v_r = \frac{1}{4} \pi d_r^2 v_r = q$, by

(8) $h'_r = \frac{C l_r}{d_r^5}$;

where C is a constant. The problem then requires us to find d_1, d_2, and d_3 so that

(9) $I = I_1 + I_2 + I_3 = A \; (d_1^2 h_1 l_1 + d_2^2 h_2 l_2 + d_3^2 h_3 l_3)$

will be a minimum subject to the condition that

(10) $h' = h'_1 + h'_2 + h'_3 = C\left(\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} \right)$

shall have a specified value. If, now λ is a constant, the expression,

(11) $L = I + \lambda h'$,

will be a minimum when I is a minimum. Since h' shall have a definite value, it may be considered a constant. Substituting the values of I and h' as given by equations (9) and (10) and rearranging, equation (11) becomes

(12) $L = (I_1 + \lambda h'_1) + (I_2 + \lambda h'_2) + (I_3 + \lambda h'_3) = L_1 + L_2 + L_3$

For an assumed mode of division of the pipe into sections, h_r and l_r are given for each section and therefore must be considered as constants. Consequently L_1 contains d_1 as the first variable. Similarly, L_2 contains d_2 as a second variable and L_3 contains d_3 as the third variable. L_1, L_2, and L_3 contain no variable in common and they are therefore independent functions. From this it follows that if their sum must be a minimum, each separately must be a minimum. That is, we must have separately

(13) $\frac{d\,L_1}{d\,(d_1)} = 0 \qquad \frac{d\,L_2}{d\,(d_2)} = 0 \quad$ and $\quad \frac{d\,L_3}{d\,(d_3)} = 0.$

Or in general [see eq. (11)],

(14) $\frac{d\,L_r}{d\,(d_r)} = \frac{d\,I_r}{d\,(d_r)} + \lambda \frac{d\,h'_r}{d\,(d_r)} = 0$

for each section of the pipe.

Since λ is a constant, yet to be determined, we may arbitrarily let

(15) $\lambda = \frac{b\,q}{8.8}$

and consider b the constant yet to be determined. Substituting in equation (14), we obtain

(16) $\frac{d\,L_r}{d\,(d_r)} = \frac{d\,I_r}{d\,(d_r)} + \frac{b\,q}{8.8} \cdot \frac{d\,h'_r}{d\,(d_r)} = 0$

for each section of the pipe.

From equations (7) and (8), it is seen that l_r is a factor of equation (16). Dividing through by l_r and performing the differentiation indicated, the resulting equation becomes identical with that derived by Prof. Mead (Water Power Engineering, page 546). Formula (1) therefore, gives us the required diameter for any section under high head and formula (2) for low head. The value of h to be used in formula (1) when applied to any section should be of course the highest head that section is under.

In the present interpretation of formulas (1) and (2), b is not to be considered as the value of a hydraulic horsepower at the wheel, but as a constant or parameter such that the total loss of head in the pipe shall be h'. Although a constant, b is not arbitrary, but its value must be determined. This can be done in the following way: For riveted steel pipes, the diameters as determined by formulas (1) and (2) must satisfy the equation [see equation (10)]

(17) $h' = C \left[\frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + \; . \; . \; . \right]$

If, therefore, the values of d_1, d_2 and d_3 as given by formulas (1) and (2) be substituted in equation (17), the resulting equation will contain b as the only unknown, and therefore b is determined. If, for instance, the first section is under low head and the other two under high head, d_1 is given by formula (2) and d_2 and d_3 by formula (1).

Instead of determining b algebraically as was suggested above, a value for b may be assumed and the diameters determined by means of the formulas. A few trials will suffice to determine b so that the total loss of head in the pipe line will be h'.

Since for a given mode of division of the pipe line into sections, h' is determined as soon as a value for b is assumed, the formulas (1) and (2) may be given a new interpretation. According to this interpretation b may be taken as a parameter such that for every assumed value for b, the resulting pipe line will, for the resulting total loss of head in the pipe, be the pipe of least annual cost (or least first cost or least amount of metal). This interpretation, so far as the writer knows, is here given for the first time.

In special cases b may be taken as the value of a hydraulic horsepower at the wheel. The new interpretation, however, still holds since the pipe as thus determined