A varies as b when a is constant.

" " a " b " ".

And $A = \frac{1}{2}ba$, and $\therefore$ varies as ab when these both vary.

6. (a): Find the number of permutations of n things taken r together.

Let there be r boxes which are to be filled by placing in each, one out of n letters. The number of ways in which this can be done is the number of permutations of n things taken r together.

In filling the first box we have n choices, as we may take any of the n letters. In filling the second box we have $n - 1$ choices amongst the $n - 1$ remaining letters, and these may be combined in every possible way with the first n choices. Therefore we can fill two boxes in $n(n - 1)$ ways.

Similarily we may fill three boxes in $n(n - 1)(n - 2)$ ways, etc.

$$\therefore \; {}^nP_r = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$$

(b). The value of ${}^nP_n$ is $n!$

Let ${}^nP(a)$ be the number of permutations of n things of which a are alike.

If a were all different, they would give rise to $a!$ permutations, each of which might be combined with each of ${}^n\Gamma(a)$, and this would give ${}^nP_n$.

$\therefore \; {}^nP(a).a! = n! \qquad \therefore \; {}^nP(a) = {}^n/_{a!}$

Similarly if a be alike of one kind, and b be alike of another kind,

$${}^nP(ab) = \frac{n!}{a!\,b!} \text{; etc.}$$

(c). In how many ways can $p + 2n$ different things be divided into three groups containing p, n, and n things respectively?

We can make a group of p things out of $p + 2n$ things in ${}^{p+2n}C_p$ ways, and for each way we have a group of $2n$ things left. These we may divide into groups of n things in ${}^{2n}C_n$ ways; but, as every group will be repeated, we must divide this by 2; and, as each of these may be combined with each group of p things, the total number must be

$$ {}^{p+2n}C_p \cdot \tfrac{1}{2} \cdot {}^{2n}C_n \text{, or } \frac{(p+2n)!}{p!\,(2n)!} \cdot \tfrac{1}{2} \cdot \frac{(2n)!}{n!\,n!} = \frac{(p+2n)!}{2\,.\,p!\,.\,(n!)^2}$$

(d). Find the number of terms in the expansion of $(a+b+c+d+e)^8$.

As every term will be homogeneous of 8 dimensions, this comes to finding the number of homogeneous terms of 8 dimensions which can be made from 5 letters and their powers.

$$\text{This is } \frac{5.6.7\,8.9.10\,11.12}{1\,2.3.4.5\,.\,6\,.\,7\,.\,8} \text{, or } 495.$$

Find the co-efficient of $a^3bc^2d^2$ from the fore-going expansion.

This may be found by using the formula of the multinominal theorem. But this theorem is so cumbrous and so little used that it is not worth remembering. It may be found directly as follows.

The co-efficient of $a^3bc^2d^2$ is the same as that of $a^3b^2c^2d$

In $(a+\overline{b+c+d+e})^8$ the term containing a^3 is ${}^8C_5(b+c+d+e)^5$. In $(b+\overline{c+d+e})^5$, the term containing b^2 is ${}^5C_3(c+\overline{d+e})^3$; and in $(c+\overline{d+e})^3$ the term containing c^2 is ${}^3C_1(d+e)$ $\therefore$ The co-efficient required is

$${}^8C_5 \,.\, {}^5C_3 \,.\, {}^3C_1 = {}^8C_3 \,.\, {}^5C_2 \,.\, {}^3C_1 = 1680.$$

(*To be continued.*)