

ily say, four dimensional space requires four independent magnitudes for specification of the position of a point, or yields a quadruple infinity of points. Or why may we not say, in four dimensional space there are four independent directions?

We may take these two sticks and easily place them perpendicular to each other, then we may take a third stick and place it perpendicular to the other two. These represent the independent directions in two and three dimensional space. Where shall we place the fourth stick for the fourth independent direction? Of course, we can not find such a position, try as we may. Does this mean that there is no such direction? If we believe the results of our experience, assuredly there is no such direction and there is no fourth dimension. Apparently this is the end of the matter. But here is where the interest in the subject begins. The fourth dimensionists, of whom there are a goodly number, and some really earnest writers too, tell us that because we can neither find nor imagine such a direction that its non-existence is not proved. Probably were it not for the fact that a large number of very interesting and some uncanny conclusions follow logically from the assumption of a fourth dimension, we would have heard little of it except in lectures on Hyperspace. But once the existence of the new direction or the fourth perpendicular is admitted, then we are free to go as far as we like. Of course this assumption leads immediately to a geometry of figures in four dimensional space as much more varied in its forms and extensive in its relations than ordinary solid geometry, as solid geometry is more extensive than plane. As in solid geometry all the facts and proofs of plane geometry are valid, so the new geometry of Hyperspace includes all that has gone before in the lower forms.

Since the introduction by Kant of the idea that space might have more dimension than three, which idea was later developed by Gauss and Lobatchewsky, there have been many writers on the subject, some of whom have contributed books and papers of the greatest interest. The great difficulty experienced by everyone is the impossibility of realization of the new direction and the fact that figures can not be visualized, at least by most of us, though we may be able to tell many of their properties. A few people have claimed an ability to think in four dimensions, but in all probability they claim merely a familiarity with figures to the extent that they may make