

It is not necessary to state that objects should be discarded as soon as it is perfectly clear that the pupils understand the meaning of abstract number. The idea is to lead pupils to the abstract through the concrete, then abandoning the latter, to return to it only when it is perceived that figures are no longer associated in the pupil's mind with clear, well defined notions of the numbers which they represent, but with words, the names of numbers.

To sum up, the following order should be adopted in teaching arithmetic to beginners: intuition; mental and oral work; written work; theoretical knowledge, limited to what is absolutely necessary, should be deduced from carefully chosen well graded examples. Definitions are given only when the operations to which they apply are already well understood. For example, to small children already able to solve problems in the simple rules, using concrete or abstract numbers, and acquainted with the signs of the operations, the teacher will simply say:  $4 + 3 = 7$ , this is an addition; of  $7 - 2 = 5$ , this is a subtraction;  $3 \times 3$  or three times three ( $3 + 3 + 3$ ) = 9, this is a multiplication; share 8 apples among four children 2 to each (or  $8 \div 4 = 2$ ), this is a division. More complete definitions should not be given till a later period.

Mental arithmetic, which is the basis of the knowledge of numbers and of the surest short cuts, should receive special attention from the teacher in every class. Pupils will, therefore, be trained from the beginning in calculating mentally, but the work should be reasoned and not done mechanically. It is well to remember that mental work deals with numbers and not with figures; were it otherwise, the teacher would have his pupils doing written work in their minds. Mental arithmetic has processes of its own which differ from those of written arithmetic. The child should be taught these processes, and it is the teacher's duty to explain them to him and to see to it that he employs them. Thus in adding 57 and 38, the pupil should, in thought, decompose the two numbers into tens and units; 50 and 30, 80, to which should be added 7 plus 8 to form 95 in all. In the same way in adding 257 and 138, the numbers should be decomposed, in the mind, into hundreds, tens, and units:  $200 + 100 = 300$ ;  $50 + 30 = 80$ ; in all 380, to which should be added  $7 + 8$  to make 395. Analogous devices should be followed in all exercises done mentally.

It is even desirable that problems of all kinds be first solved mentally, small numbers being used for this purpose, before being given to be worked out in writing, with large numbers.

But the first and most important point is that new elements be presented gradually and progressively, that they be accompanied by numerous applications which shall cause them to sink into the mind and to fix themselves in the memory in a lasting manner. At this point a few examples will not be out of place. It is desired to teach intuitively the abstract truth that 4 and 4 make 8: the pupils are shown 4 marbles and 4 marbles, 4 pencils and 4 pencils, 4 points (:) and 4 points (:) perceiving that,