III. DISCOUNT.

D = Discount; other symbols as before.

$D = A - P =$ from (6) and (7) $\frac{A(1+rt)}{1+rt} - \frac{A}{1+rt} = \frac{Art}{1+rt}$ (13);

$\therefore$ Present Worth or $P = \frac{A}{1+rt}$ (14).

IV. COMPOUND INTEREST.

Since £1 at the end of 1st year amounts to $1 + r$.

$1 : 1 + r :: P : P(1 + r) =$ Amount of P at the end of the 1st year.

$1 : 1 + r :: P(1 + r) : P(1 + r)^2 =$ " ....2nd "

$1 : 1 + r :: P(1 + r)^2 : P(1 + r)^3 =$ " ....3rd "

And so on, therefore at the end of the tth year $A = P(1+r)^t$ (15);

$\therefore P = \frac{A}{(1+r)^t}$ (16); $r = \sqrt[t]{\left(\frac{A}{P}\right)} - 1$ (17); $t = \frac{\log. A - \log. P}{\log. (1+r)}$ (18). From (18) the time in which any sum, as S, will amount to n times S at Compound Interest is represented by $t = \frac{\log. n.}{\log. (1+r)}$ (19).

V. ARITHMETICAL PROGRESSION.

Let a = first term, l = last term, d = common difference, n = number of terms, and S = sum of the series.

$S = a + (a + d) + (a + 2d) + (a + 3d) + \ldots\ldots (l - d) + l$, Reversing the series.

$S = l + (l - d) + (l - 2d) + (l - 3d) + \ldots (a + d) + a$ Adding.

$2S = (a + l) + (a + l) + (a + l) + (a + l) \ldots (a + l)$ to n terms $= n(a + l)$.

$\therefore S = \frac{n}{2}(a + l)$ (20); $a = \frac{2S}{n} - l$ (21); $l = \frac{2S}{n} - a$ (22);