(as these are ordinarily designed), if any, which lies above the level of the crown—in fact to all of the arch system at the crown. The rest of the arch dead load varies from zero at the crown, C, to a definite amount at the skewbacks, S, S′. Let us call these parts I. and II. Let the area M M′ T T′ represent I, and the area S C S′ M M′ represent II. The weight of the spandrel supports (or that part of them in II) will vary sensibly as the ordinates between M M and the curve. We must assume that the spandrel supports are of uniform section and spaced equally, and that the floor system is of uniform weight per horizontal foot, in accordance with usual practice. Let us further assume for the present that that part of the ring which varies from zero at C to any amount we wish at S varies according to the same law. Here f represents the actual rise of the linear arch, but $y_0$ is, of course, not the actual height of the floor system. Its meaning is not of practical interest. It is the height to which the part of the arch in I would rise if it weighed the same per cubic foot as the part in II, the latter supposed to be distributed uniformly over M M′ S C S′.

We may suppose the dead weight to be thus distributed, in which case let it weigh $\omega$ lbs. per cubic foot.

Assume that the linear arch S C S′ is the equilibrium curve and also the middle line of the arch ring. Then we find as follows:—

The last term of IV is an hyperbolic function, cosh $\frac{x}{a}$ Tables of these functions were prepared by the Smithsonian Institution and published in 1909.* We shall have recourse to them in what follows.

The equation of the curve then becomes—

$$\frac{y}{y_0} = \cosh\frac{x}{a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{V.}$$

We have—

$\omega y_s$ = weight per lin. ft. of whole arch system at S = S (say) which must be computed, .... VI.

$\omega y_0$ = weight per lin. ft. of arch system at C = C (say) which must be computed ...... VII.

When $x = \frac{l}{2}$, $y = y_s$.

$$\therefore \frac{y_s}{y_0} = \cosh\frac{l}{2a} = \frac{S}{C} \text{(from VI. and VII.).}$$

whence a is found from the tables.

Also—

$$\frac{S}{C} = \frac{y_s}{y_0} = \frac{f}{y_0} + 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{VIII.}$$

whence $y_0$ from VIII, $\omega$ from VII and H from I.

We are now ready to tabulate the values of x for as many points on the curve as is required, say the points at the feet of the spandrel posts, or 8 or 10 points for the half arch. Then tabulate $\frac{x}{a}$, cosh $\frac{x}{a}$ and $y_0$ cosh $\frac{x}{a}$ which gives the values of y. It will now be convenient to subtract $y_0$ from each value of y giving the ordinates $y - y_0$. We then have the abscissae and ordinates of the curve conveniently on a horizontal line through the crown of the linear arch and perpendicular to it.

Let us now examine the assumptions made above as to distribution of weight. In Fig. 2 the central part of the ring is that part which is of uniform weight per horizontal foot, part 1. The shaded portions are part 2 of the ring, varying from zero at C to anything required at S. Also any one spandrel post is shown, the shaded part being part 2 of it. The shaded parts of the rib have been assumed to vary in weight per horizontal foot from C to S according to the ordinates $y - y_0$. Suppose the rib to have been designed as to intermediate depths between C and S in this way, then we prove as follows:—

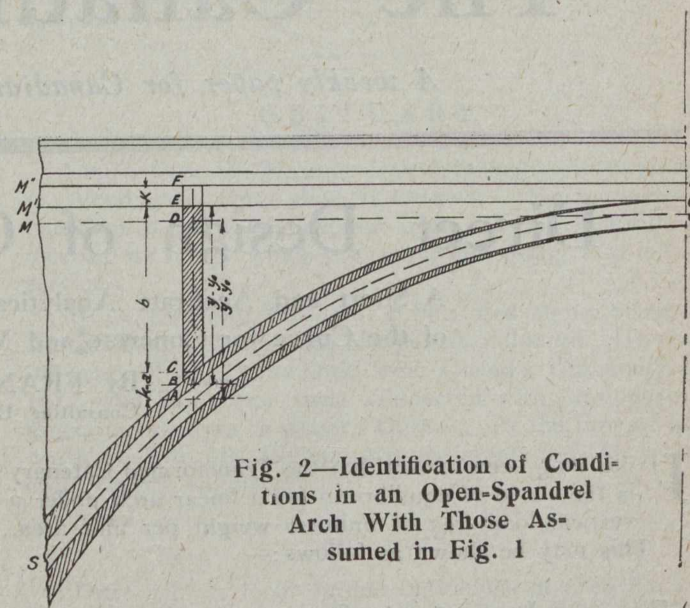


**Fig. 2—Identification of Conditions in an Open-Spandrel Arch With Those Assumed in Fig. 1**

A B = C C′, = D E ........................(a)

Since the weight of this part of the rib is uniform per horizontal foot.

$\therefore$ B E = A D = $y - y_0$ ........................(b)

Also B C varies as $y - y_0$ ........................(c)

Let B C = a,

C E = b.

Let similar parts of any other post be a′, b′.

Then $\frac{a + b}{a' + b'} = \frac{y - y_0}{y' - y_0}$ = from (b),

= k, say.

Also $\frac{a}{a'}$ = k from (c).

$$\therefore \frac{a'k + b}{a' + b'} = k$$

whence $\frac{b}{b'} = k$

or C E varies as $y - y_0$.

Now C E is all that part of the post in division II, as the part (if any) above it is the part above the extrados of the ring and is of uniform weight per horizontal foot if the spandrel posts are equally spaced and of uniform section.

We have now shown that all that part of the bridge which is not of uniform weight per horizontal foot varies from zero at the crown to any required weight at the skewback, directly as the ordinates $y - y_0$, as required by the theory, but we must design the intermediate depths of the arch ring accordingly, although we may choose any depth at the crown and any other at the skewback. When the depths of the latter sections are so chosen as to make the same maximum stresses at both crown and skewback then the ring with intermediate depths chosen as above is almost ideally correct for arches with fixed ends, judging by various arches analyzed by the writer, but is slightly in excess of the requirements along the middle haunch where temperature moment stresses are small. But if the arch of ideal depth at all points is sought and found by repeated trials it will not differ at all from the one here assumed at the crown or skewback, and only so slightly along the haunches as to make no

*Smithsonian Mathematical Tables. Hyperbolic Functions. Published by the Smithsonian Institution, Washington.