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COLUMNS.†

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In the whole range of subjects with which the art of engineering is concerned, there are few in which theory has contributed less to the advance of intelligent practice than in this. The reason cannot be found in any want of importance in the subject itself, for, of the materials used in construction, a much greater part is used in compression than in tension, and a very considerable part is used in the form of columns or struts. Until recently too, the experiments on compressive strength were very few in number and narrow in range, compared with the multitude of tensile experiments that have been made. Even now, many of the laboratories with expensive and elaborate apparatus for tensile tests have no means of making compressive tests. Of late years, however, American experimenters have done a great deal to remedy this state of things, among whom Mr. Christie, Mr. Bouscaren, and Clarke, Reeves & Co. may be mentioned in particular.

What is now wanted even more than experiments, is a theory of the subject which will enable us to interpret their results in a general way, and also to indicate the kind of experiments desirable. We shall therefore attempt here to give a treatment of the subject somewhat more adequate than is usually found in text books.

A column differs from a beam or girder in the nature of the forces it is intended to resist, a beam being used to resist forces transverse to its length, a column to resist forces directed between its two ends. Of course, the same material may be used to resist forces of both kinds at once, but for the sake of clearness we shall neglect transverse forces altogether in treating of the column. The leading feature in the behaviour of a loaded column is that it bends, the deflection constantly increasing with the load. The stress which might at first be almost uniformly distributed in the column, becomes thus more and more unequal at the outer and inner edges as the load increases. With a tensile stress on the other hand, although a small flexure undoubtedly must result from the same causes that give rise to flexure of columns, the direction of the flexure is opposite, and therefore tends to produce a more uniform distribution of stress. The difference between the two cases may be illustrated by that between stable and unstable equilibrium.

The equation which forms the basis of all investigations into the flexure of beams and columns is

$$(1) \qquad M = EI\left(\frac{1}{R} - \frac{1}{R_o}\right)^*$$

where M moment of external forces at a cross section of the beam or column, whose moment of inertia about an axis through its centre of gravity at right angles to the plane of flexure is I;

R = radius of curvature of axis of beam or column at same cross section, and R_o = initial value of R when M = 0

E = modulus of elasticity of material.

By the axis of a column we here and hereafter mean the axis of figure and not the neutral axis (which for columns generally lies at a considerable distance outside the column).

It will be well here to call attention to the assumptions on which (1) rests, because it has often been applied to circumstances in which it

† This investigation coincides in certain parts with papers by Professors Ayrton and Perry, in the *Engineer* (London), December 10th and 24th, 1886, and by Professor Krohn in the Proc. Am. Soc. C.E., 1887.

* This equation is usually given as $M = \frac{EI}{R}$, thus assuming the initial form of the axis to be a straight line. For a beam, although the assumption cannot be true in fact, it does not affect the distribution of stress in the beam. With a column, however, it is of vital importance, as we shall see.