may be necessary, as we do in arithmetic and algebra. Certainly the usual way of arriving at the above propositions would correspond to making the figure of and proving, say, the 47th of Book I. every time we wished to use it, or, to use a more direct illustration, to establishing or illustrating the distributive law of algebra every time we wished to apply it.

University of Toronto.

ANNUAL EXAMINATIONS, JUNE, 1879.

JUNIOR MATRICULATION.

MATHEMATICS.

Pass Paper.

Examiner: F. Hayter, B.A.

1. Define the Greatest Common Measure and Least Common Multiple of any number of quantities. How is the L. C. M. of a number of fractions found?

Add together $\frac{13}{42}, \frac{59}{63}, \frac{83}{121}, \frac{3}{70}, \frac{91}{110}, \frac{91}{264}$.

2. Prove the rule for the conversion of a circulating decimal into a vulgar fraction, using a numerical example.

3. Distinguish between interest and discount, and shew that if P, I, D, be respectively the principal sum, and the interest and discount upon it for any given time.

$$\frac{1}{D} = \frac{1}{I} + \frac{1}{P}.$$

4. A person has an income derived from £3360, which was originally invested in the Four per cents at 96. If he now sells out at 94, and invests one half of the proceeds in Railway Stock at $82\frac{1}{4}$, which pays a dividend of 3 per cent., and the other half in Bank Stock at $164\frac{1}{2}$, paying $8\frac{1}{2}$ per cent. dividend, what difference will he find in his income?

5. Simplify

(i) $\dfrac{2^{n+4}-2\times 2^n}{2^{n+2}\times 4}$ (ii) $\dfrac{x^2+\left(\frac{a}{b}+\frac{b}{a}\right)xy+y^2}{x^2+\left(\frac{a}{b}-\frac{b}{a}\right)xy-y^2}$

(iii) $\dfrac{\frac{a^2+b^2}{b}-a}{\frac{1}{b}-\frac{1}{a}} \times \dfrac{a^2-b^2}{a^3+b^3} \times \left(\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}\right) \times \left(\dfrac{a}{a+b}+\dfrac{b}{a-b}\right)$

6. Divide $6x^6-4x^4-19x^3+23x^2-13x+3$ by $3x^2-2x+1$, (i) in full; (ii) by Horner's method.

7. Prove the rule for finding the G. C. M. of two quantities. Find the G. C. M. of $(x^3+x^2y-3xy^2+y^3)$ and $(x^3+3x^2y+xy^2-y^3)$.

8. Solve

(i) $\dfrac{3-x}{2+x}-\dfrac{2-x}{2+x}+\dfrac{1-x}{1+x}=1$. (ii) $x^2+4{\cdot}8x+2{\cdot}87=0$.

(iii) $\sqrt{2}+1-(2^{\frac{1}{x}}-1)^{-1}=0$.

9. Extract the square root of $32+10\sqrt{7}$.

10. Solve

(i) $\begin{cases} x+y=a \\ x^4+y^4=14x^2y^2 \end{cases}$ (ii) $\begin{cases} \dfrac{(x+y)^2}{a^2}+\dfrac{(x-y)^2}{b^2}=8 \\ x^2+y^2=2(a^2+b^2) \end{cases}$

(iii) $\begin{cases} (x+y)(x^3+y^3)=1216 \\ x^2+xy+y^2=49 \end{cases}$ (iv) $\begin{cases} x^2yz=a \\ y^2zx=b \\ z^2xy=c \end{cases}$

11. If a side of any triangle be produced, the exterior angle is equal to the two interior and opposite angles; and the three interior angles of every triangle are together equal to two right angles.

The difference of the angles at the base of any triangle is double the angle contained by a line drawn from the vertex perpendicular to the base, and another bisecting the angle at the vertex.

12. To describe a parallelogram that shall be equal to a given triangle, and have one of its angles equal to a given rectilineal angle.

13. The opposite angles of any quadrilateral figure inscribed in a circle are together equal to two right angles.

If two opposite sides of a quadrilateral figure inscribed in a circle be equal, prove that the other two are parallel.

RESULTS.

1. Book work. 2. Book work.

3. $Prt = I$, $\frac{Prt}{1+rt} = D$; $\therefore \dfrac{I}{1+\frac{I}{P}} = D$, or $\frac{1}{D} = \frac{1}{I} + \frac{1}{P}$.

4. £5. 5. (1) $\frac{7}{8}$. (2) $= \dfrac{(ax+by)(bx+ay)}{(ax-by)(bx+ay)} = \dfrac{ax+by}{ax-by}$.

(3) $2a\dfrac{(a^2+b^2)^2}{(a^2-b^2)^2}$. 6. $2x^3-7x+3$. 7. Book work. $x^2+2xy-y^2$.

8. (1) 0 or $-2\pm\sqrt{-1}$. (2) $-{\cdot}5$ or $-4{\cdot}3$. (3) Equation reduces to $2^{\frac{1}{2}}\left(1+2^{\frac{1}{2}}\right) = 2^{\frac{1}{x}}\left(1+2^{\frac{1}{x}}\right)$, or $x=2$. 9. $5+\sqrt{7}$.

10. (1) From 2nd equation $x^4+2x^2y^2+y^4 = 16x^2y^2$; $\therefore x^2+y^2 = \pm 4xy =$, from eq. 1, a^2-2xy, $\therefore xy = \frac{a^2}{6}$, or $-\frac{a^2}{2}$; and substitute for either x and y in 1. (2) From 1st eq., $8a^2b^2+2xy(a^2-b^2) = (x^2+y^2)(a^2+b^2) = 2(a^2+b^2)^2$ by 2nd eq.; $\therefore xy = a^2-b^2$, and $\therefore$ by 2nd equation, $x+y = \pm 2a$, and $\therefore$ by 1st eq., $x-y = \pm 2b$, &c. (3) Dividing 1st eq. by 2nd, $x^2+2xy+y^2 = \frac{1216}{49}$, then by 2nd eq., $xy = -\frac{1185}{49}$, and $\therefore$ by 2nd, $x^2-2xy+y^2 = \frac{5956}{49}$; $\therefore x+y = \pm\frac{8}{7}\sqrt{19}$, $x-y = \pm\frac{2}{7}\sqrt{1489}$, &c. (4) Multiplying the equations $x^4y^4z^4 = abc$, or $xyz = \sqrt[4]{abc}$, $\therefore$ from first equation $x = \sqrt[4]{\frac{a^3}{bc}}$, &c.

11. Let ABC be the triangle, AD perpendicular to BC, and AE bisecting BAC. Then $B-C = AEC-AED = EAD+ADE-AED = 2EAD+AED-AED = 2EAD$.

13. Let $ABCD$ be the quad., having $AB = DC$. Then because the arcs on which they stand are equal angle $ADB =$ angle DBC; $\therefore AD$ is parallel to BC.

ALGEBRA.

HONORS.

Examiner—A. K. Blackadar, B.A.

1. Define a *fraction*, and prove that

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

Simplify

$$\frac{\frac{1}{1-a}-\frac{1}{1-b}}{\frac{1}{(1-a)b}-\frac{1}{(1-b)a}} \times \frac{\frac{1}{1-b}-\frac{1}{1-c}}{\frac{1}{(1-b)c}-\frac{1}{(1-c)b}} \times \frac{\frac{1}{1-c}-\frac{1}{1-a}}{\frac{1}{(1-c)a}-\frac{1}{(1-a)c}}$$

2. Describe methods of finding the G.C.M. of two algebraical quantities.

Show that $(a-b)(b-c)(c-a)$ is the G. C. M. of $(a+b)(a-b)^3+(b+c)(b-c)^3+(c+a)(c-a)^3$ and $(a-b)(a+b)^2+(b-c)(b+c)^2+(c-a)(c+a)^2$.

Find also the least common multiple of these two quantities.

3. Find the square root and the fourth root of

$$x+x^{-1}-4\sqrt{-1}\,(x^{\frac{1}{2}}-x^{-\frac{1}{2}})-6.$$

If $x^4+2ax^3+bx^2+2cx+d$ is a complete square, prove that $a = \dfrac{c}{\sqrt{d}} = \dfrac{b-2\sqrt{d}}{a}$.

4. Find the roots of the equation $ax^2+bx+c=0$.

What do the roots become when (1) $a=0$; (2) $c=0$; (3) $a=0$ and $b=0$?

Prove that a quadratic equation can have only *two* roots.