(3) $x - y = a$ (1) (1)+(2), $x - z = a + b$,
$y - z = b$ (2) $x + z = c$
$z + x = c$ (3) $2x = a + b + c$

$$x = \frac{a+b+c}{2},\ y = \frac{b+c-a}{2},\ z = \frac{c-a-b}{2}$$

7 (1) Solve the equation $ax^2 + bx + c = 0$, and interpret your result according as $a = 0$, or $b = 0$, or $a = b = 0$.

(2) If $a + b + c = 0$, find values of x that will satisfy

$$\frac{a}{x+b} + \frac{b}{x+c} + \frac{c}{x+d} = 0.$$

7. (1) Bookwork.

(2) $\dfrac{a}{x+b} + \dfrac{b}{x+c} + \dfrac{c}{x+d} = 0.$

$$a(x+c)(x+d) + b(x+b)(x+d) + c(x+b)(x+c) = 0,$$

$$x^2(a+b+c) + x(ac+ad+b^2+bd+c^2+bc) + acd+b^2d+bc^2 = 0,$$

$$x\{c(a+b+c) + b^2 + bd + ad\} = -(acd + b^2d + bc^2),$$

$$x = -\frac{acd + b^2d + bc^2}{b^2 + bd + ad}.$$

8 If α, β be roots of $ax^2 + bx + c = 0$, and $\alpha + \beta$, $\alpha - \beta$ roots of $a'x^2 + b'x + c' = 0$, show that $ab'^2 - 2a'bb' + 4a'^2c = 0$.

$$\alpha + \beta = -\frac{b}{a} \qquad (\alpha+\beta) + (\alpha-\beta) = -\frac{b'}{a'},$$

$$\alpha\beta = \frac{c}{a} \qquad -\frac{b}{a} + \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}} = -\frac{b'}{a'},$$

$$\frac{b'}{a'} - \frac{b}{a} = \sqrt{\frac{b^2 - 4ac}{a^2}},$$

$$ab' - a'b = a'\sqrt{(b^2 - 4ac)},$$

$$a^2b'^2 - 2aa'bb' + a = a' - 4a'^2ac,$$

$$\therefore ab'^2 - 2a'bb' + 4a'^2c = 0.$$

9. The sides of a box are all rectangles, and the areas of the unequal sides are $7\frac{1}{2}$, 15 and $4\frac{1}{2}$ Find the lengths of the sides.

9 Let x, y, z, be edges,

$$xy = 7\tfrac{1}{2} \quad (1)$$
$$xz = 15 \quad (2)$$
$$yz = 4\tfrac{1}{2} \quad (3)$$

$$(1) \times (2) \div (3)\ x^2 yz = \frac{15 \times 15}{2}$$

$$x^2 = 25,\ x = 5,\ y = 1\tfrac{1}{2},\ z = 3.$$

Intermediate and Third Class

ALGEBRA.

Solutions by Wilbur Grant, C.I., Toronto.

I. Divide

(1) $(a-b)c^3 + (b-c)a^3 + (c-a)b^3$ by $(a-b)(b-c)(c-a)$.

(2) $\dfrac{x^2+y^2}{x^3y^2} - \dfrac{x^2+y^2}{x^2y^3}$ by $\dfrac{1}{x} - \dfrac{1}{y}$.

Ans. (1) $-(a+b+c)$.

(2) $\dfrac{x^2+y^2}{x^3y^3}$.

II. What must be the values of a, b and c that $x^3 + ax^2 + bx + c$ may have $x - 1$, $x - 2$ and $x - 3$, all as factors?

Ans. $a = -6$, $b = 11$, $c = -6$

III. Find the H C.F. of

(1) $3x^4 - 4x^3 + 1$ and $4x^4 - 5x^3 - x^2 + x + 1$.

(2) $8x^3 - y^3 + 27z^3 + 18xyz$ and $4x^2 + 12xz + 9z^2 - y^2$.

Ans. (1) $(x-1)^2$. (2) $2x + 3z - y$.

IV. Simplify

(1) $\left(\dfrac{4x^2}{y^2} - 1\right)\left(\dfrac{2x}{2x-y} - 1\right) + \left(\dfrac{8x^3}{y^3} - 1\right)\left(\dfrac{4x^2+2xy}{4x^2+2xy+y^2} - 1\right)$.

(2) $\dfrac{x^3 + (a+b)x^2 + (ab+1)x + b}{bx^3 + (ab+1)x^2 + (a+b)x + 1}$.

Ans. (1) 2. (2) $\dfrac{x+b}{bx+1}$.

V. Find the value of x that will make $\dfrac{ac+bd+ad+bc}{x-3c+2d}$ independent of c and d.

Ans. $-5d$.

VI. (1) If $a + b + c = 0$, then

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right\}^2.$$

(2) If $x = a^2 + b^2 + c^2$ and $y = ab + bc + ca$, then $x^3 + 2y^3 - 3xy^2 = (a^3 + b^3 + c^3 - 3abc)^2$.

(3) If $2a = y + z$, $2b = z + x$, $2c = x + y$, express $(a+b+c)^3 - 2(a+b+c)(a^2+b^2+c^2)$ in terms of x, y and z.

Ans. (1) Taking $a + b + c = 0$, then

$$\frac{2(a+b+c)}{abc} = 0,\ \frac{2}{bc} + \frac{2}{ac} + \frac{2}{ab} = 0,$$

$$\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{2}{bc} + \frac{2}{ac} + \frac{2}{ab}\right) - \left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) = 0,$$