STRESSES IN MASONRY DAMS.*

By William Cain, M. Am. Soc. C.E.

The object of this investigation is to determine the amounts and distribution of the stresses in a masonry dam, at points not too near the foundations, having assumed the usual "law of the trapezoid," that vertical unit pressures on horizontal planes vary uniformly from face to face.

Experiment indicates that such vertical stresses increase pretty regularly in going from the inner to the outer face, for reservoir full, until we near the down-stream or outer face, where the stress gradually changes to a decreasing one, which decrease continues to the end of the horizontal section. The law of the trapezoid is thus only approximately true over part of the section, but, as it gives an excess pressure where it attains a maximum, it errs on the safe side.

The profile of the dam selected is of the triangular type, with some additions at the top, but the method used in determining the stresses is general, and will apply to any type of profile. The final equations will give at any (interior or exterior) point of horizontal section considered the vertical unit stress on the horizontal section, the normal stress on a vertical plane, and the unit shear on either horizontal or vertical planes. From these stresses, the maximum and

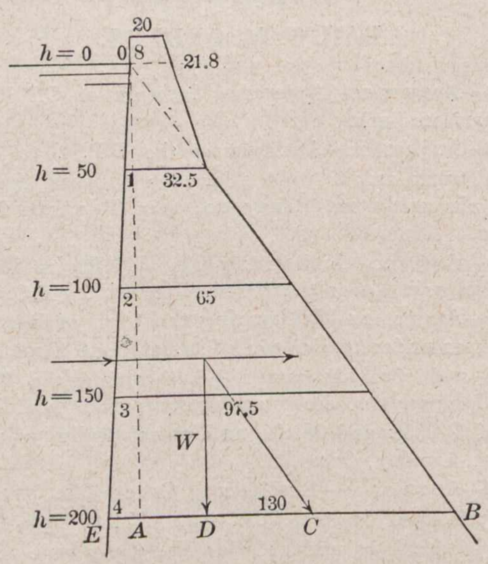

Fig. 1.

minimum normal stresses, and the planes on which they act, can be determined, and ultimately, if desired, the stress on any assumed plane can be ascertained.

The solution presented is approximate, which is justifiable in view of the approximation involved in "the law of the trapezoid" used. The results, however, are practically correct, as will be evident from the checks applied, resulting from the exact theory given in the Appendix. The theory used, being simple, should be easily followed.

Let Fig. 1 represent a slice of the dam contained between two vertical parallel planes, 1 ft. apart and perpendicular to the faces. The batter of O B is, $\frac{130}{200} = \frac{0.65}{1}$; that of O E being $\frac{4}{200} = \frac{0.02}{1}$. The batter of the inner face was found by trial, so that the centers of pressure on horizontal sections, for reservoir empty, should nowhere pass more than a fraction of a foot outside the middle third of the section. The simple type of profile shown was adopted for ease of computation.

For convenience in subsequent computations, the breadth, b = E B, of horizontal sections, corresponding to various depths, h, below the surface of the water in the reservoir, are given, all dimensions being in feet:

h=199.0, b=133.330
h=199.5, b=133,665
h=200.0, b=134.000
h=200.5, b=134,335
h=201.0, b=134,670

Take the weight of 1 cu. ft. of masonry equal to 1; then the weight of masonry above any section is equal to the corresponding area in Fig. 1 above that section. The area of the portion above E O B is readily found to be 712, and its moment about the vertical, A O, is 11 603, the unit of length being the foot. In Fig. 1, D is where the vertical through the centre of gravity of the dam above the joint, E B, cuts that joint, and C is the centre of pressure on that joint when the water pressure on E O is combined with the weight of masonry, W, above E B.

As h varies, suppose each horizontal joint, in turn, marked similarly to the joint at h=200, with the letters E, A, D, C, B; then, for any joint, on taking moments of the triangles, A O B, A O E, and the area above O B, we find,

$$A\,D = \frac{\frac{A\,O}{6}(A\,B^2 - E\,A^2) + 11\,603}{W}$$

Assuming that the masonry weighs $2\frac{1}{2}$ times the water per cubic unit, then the weight of a cubic foot of water is $\frac{2}{5}$. It would entail but little extra trouble here, where the inner face has a uniform batter throughout, to include the vertical component of the water pressure on the face, E O; but it will be neglected as usual.

The horizontal water pressure for the height, h, is thus, $\frac{2}{5} \times \frac{h^2}{2} = \frac{1}{5}h^2$, and its moment about C is, $\frac{1}{5}h^2 \times \frac{1}{3}h = \frac{1}{15}h^3$.

Taking moments of W and water pressure about C, we have at once,

$$D\,C = \frac{1}{15} \times \frac{h^3}{W}.$$

From the last two formulae, we derive the following results:

h	W	A D	D C
199	13 978.335	40.49141	37.58483
200	14 112.000	40.70316	37.79289
201	14 246.335	40.91488	38.00089

A seven-place logarithmic table was used throughout, the aim in the computations being to get the seventh significant figure correct within one or two units. The necessity for this accuracy will be seen later.

The distances, E C and C B, are now readily derived.

For, h = 199, E C = 82.05624, C B = 51.27376
h = 200, E C = 82.49605, C B = 51.50395
h = 201, E C = 82.93577, C B = 51.73423.

On any plane, E B, the vertical unit pressure,

$$\text{at } B = p_1 = \frac{4b - 6\,C\,B}{b^2}\,W,$$

$$\text{at } E = p_2 = \frac{4b - 6\,E\,C}{b^2}\,W;$$

where b = E B, and W is the weight of masonry above the plane. This follows from the assumed "law of the trapezoid."

From these formulae we derive,

At h = 199, p_1 = 177.45483, p_2 = 32.22542,
h = 200, p_1 = 178.3855, p_2 = 32.24139,
h = 201, p_1 = 179.3160, p_2 = 32.25798.

Call p the vertical unit stress at a distance, x′, from E; then,

$$p = p_2 + \frac{p_1 - p_2}{b}\,x';$$

and the total stress on the base, x′, is,

$$P = \frac{1}{2}[p_2 + p]\,x' = p_2 x' + \frac{p_1 - p_2}{2b}\,x'^2 \quad \ldots\ldots\ldots\ldots(1)$$

---

*Proceedings of the American Society of Civil Engineers Vol. XXXV. Page 38, not read before the Society.