THE EXPRESSION OF PIVOT ERRORS BY A FORMULA.*

By R. Meldrum Stewart,
Dominion Observatory, Ottawa.

EVEN the best measurements of pivot errors of a meridian circle or transit instrument are, of course, affected by accidental errors, and in the case of fairly good pivots these are no doubt larger than the actual deviations of the pivots from a smooth curve. It then becomes a question what pivot corrections should be adopted, both for the zenith distances at which the errors have been observed, and for intermediate zenith distances. The practice ordinarily followed is to plot the observed values and draw a smooth curve representing them as closely as possible. The objection to this proceeding is that it is not a definite one, and, what is perhaps more serious, that the amount of smoothing does not, as it should, depend on the accuracy of the original observations (for it is manifest that accurate observations should be smoothed less than inaccurate ones). It has seemed, therefore, that it would be advantageous, if possible, to obtain from the observations a definite formula which might be used in place of the smoothed curve, and from this formula to construct tables showing the zenith distances at which the pivot corrections change from unit to unit. In a recent series of measurements at the Dominion Observatory it was found that a good representation could be obtained by the use of a few terms of a Fourier series, and it seems probable that the values so adopted are more accurate than the actual observed values.

It is evident that a Fourier series can be made to represent the observed values to any required degree of accuracy; for example, in the case where the pivot errors are observed at intervals of 6°, the use of 72 terms would reproduce the observed values exactly. Since, however, these observed values contain errors of measurement, it is probable that a more exact representation of the actual pivot errors will be obtained by omitting the terms with small co-efficients; it then becomes a question simply of the number of terms to be retained.

It is evident that, the more accurate the original observations, the more closely the adopted curve should be made to represent them. Also, the accuracy of the observations is represented by the probable error of a single observation, computed in the usual way; since several complete sets of observations are always made, this is readily obtained. Hence the deviations of the adopted curve from the observed values should be a function of the observed probable error. But by treating these deviations as residuals they themselves may be utilized to form another probable error, which represents the accuracy with which the adopted curve reproduces the observed values. The most obvious assumption is that these two probable errors should be made equal that is, that sufficient terms should be included in the Fourier series (choosing always the largest terms) to make its representation of the observed values equally accurate with the representation of the true values by the observations themselves.

Freed from technicalities, the above process simply amounts to drawing a smoothed curve in a particular way, with the adoption of a criterion for the amount of smoothing. The tentative adoption of a particular number of terms amounts to drawing a trial curve; if upon examination it is found that it is an even chance that the discrepancies between the curve and the observed values are due to errors of observation, the curve is adopted; if not, another trial curve is formed, and the process repeated until the desired result is attained.

The statement has been made that the criterion adopted (that of equality between the two probable errors) is incorrect, and a different one has been suggested, viz., that sufficient terms of the Fourier series should be included to make the probable error deduced therefrom a minimum. It seems to the writer, however, that a perusal of the preceding paragraph will establish the fact that his criterion is essentially logical and reasonable. The suggested criterion, not to mention the fact that it takes no account of the known accuracy of the observations (which can be measured independently), and that the whole principle involved in its application (that of "minimum probable error") is at least very seriously open to question, is manifestly entirely inapplicable in the present case. For it may be shown that in the attempt to represent any such observed series of quantities by a Fourier expansion, the computed probable error diminishes continually as the number of terms included increases; the proof of this theorem is elementary. Thus the adoption of this criterion would necessarily be synonymous with the use of a curve totally unsmoothed, that is, of the actual observed values.

It has been mentioned above that the use of the Fourier series amounts to drawing a smoothed curve in a particular way. It may be conceivable (though it does not appear in any way certain) that in some special case this particular way might not be suitable, i.e., that a limited Fourier series might not be applicable at all. This could probably occur only in the case of very irregular pivots, where the majority of the co-efficients in the complete Fourier series would be comparatively large. Even in this case the only fact that seems clear is that the number of terms necessary to give a satisfactory representation would be so large that the labor of computation would be prohibitive. In any case an examination of the residuals from the adopted curve will readily show whether any marked runs occur; if not the formula may be safely adopted. In the writer's judgment the occurrence of marked runs in the residuals would simply indicate that the pivots were decidedly irregular, and stood in need of re-grinding.

The actual determination of the pivot errors of the meridian circle at this observatory, referred to at the outset, was made by the microscopic method. Eight complete measurements of pivot errors were made; the probable error of a pair of microscope pointings (treated as a single observation) was found to be .0015 sec.; five cosine terms and five sine terms of the Fourier expansion (which may be combined into five cosine terms) were found to be sufficient to reduce the computed probable error to the same value. When the results are put in the form of a correction to the observed collimation it may be shown that terms involving the sine and cosine of the zenith distance have no effect; consequently these were omitted.

The resulting formula is:

$$\Delta c = {}^{s}.0010 \cos(2\theta - 188^\circ 29') + {}^{s}.0117 \cos(3\theta - 3^\circ 17') + {}^{s}.0021 \cos(4\theta - 59^\circ 45') + {}^{s}.0008 \cos(5\theta - 121^\circ 58')$$

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