in weight of 2% by waste in handling. How much less cash will he receive than he expected?

NOTE.—$\left.\begin{array}{l}\frac{5}{8}\text{ tea @ }82\\ \frac{3}{8}\text{ tea @ }65\end{array}\right\} = \text{tea @ }82\frac{1}{2} = \frac{13}{10}\,(\text{tea} \times \text{cost})$

$\therefore\ 82\frac{1}{2}$c. $= \frac{13}{10}$ cost per lb, and cost per lb $= 63\frac{6}{13}$c.

Gain on whole @ 82 would have been \$228.

Gain on 1lb $= 82 - 63\frac{6}{13} = 18\frac{7}{13}$c., $\therefore$ number of pounds bought $=$ \$228 $\div\ 18\frac{7}{13}$c. $=$ &c.

6. Reduce to its simplest form $\frac{\sqrt[3]{51}}{119}$ of $\frac{\sqrt[3]{360}}{441}$ of $\frac{\sqrt[3]{225}}{343}$.

Expression

$$=\frac{(\sqrt[3]{3}\times\sqrt[3]{17})\times(2\times\sqrt[3]{5}\times\sqrt[3]{3}\times\sqrt[3]{3})\times\sqrt[3]{5}\times\sqrt[3]{5}\times\sqrt[3]{3}\times\sqrt[3]{3}}{7\times17\times7\times3\times7\times3\times7\times7\times7}$$

$$=\frac{3\times2\times5\times\sqrt[3]{17}\times\sqrt[3]{3}\times\sqrt[3]{3}}{7^6\times17\times3\times3}$$

$$=\frac{10}{7^6\times17^{\frac{2}{3}}\times3^{\frac{4}{3}}}.$$

7. My watch was right at noon. In the evening, looking at a distant clock, I was unable to distinguish whether the clock showed five minutes to ten or ten minutes to eleven, my watch then being at twenty minutes past ten. After an hour or so, on looking at the clock, I was again unable to tell whether it pointed to eleven or to five minutes to twelve, and my watch was then at half-past eleven. What was the least possible error of the clock at the previous noon, supposing the rates of watch and clock to be uniform, and could I draw any inference as to the true time?

At first the clock is either 25 minutes slower or else 30 minutes faster than the watch. Afterwards the clock is either 35 minutes slower or else 25 minutes faster than the watch. So the clock either lost 10 minutes or lost 5 minutes while the watch went 70 minutes.

Time from noon to half-past eleven $= 690$ minutes by the watch, during which the clock must have lost $\frac{69}{7} \times 10$ or $\frac{69}{7} \times 5$ minutes, $\therefore$ least error $=$ &c.

8. If the hour and minute hands of a clock are exactly alike, show that their position will always enable us to distinguish between them except after every interval of $5\frac{7}{143}$ minutes starting from noon, and the time by the clock will then be ambiguous except after every thirteenth interval.

9. Prove the following rule for computing interest at 6% per annum for a period of months and days:—

Multiply the number of months by 5, and add $\frac{1}{6}$ the number of days; multiply this sum by the principal expressed in dollars; the result will be the interest expressed in mills.

6c. $=$ int. of \$1 for 12 mos., $\therefore$ 5 mills $=$ int. for 1 month.

6c. $=$ 60 mills $=$ int. for 360 dys., $\therefore \frac{1}{6}$ mill $=$ int. for 1 day.

$\therefore$ 5 $\times$ months $+ \frac{1}{6} \times$ days $=$ int.

10. By the Canadian Statute it is provided that the silver coins of the Canadian currency shall bear the same relation to the pound currency that the sterling silver coins bear to the pound sterling, being also of the same standard of fineness. Sterling silver is 92·5 per cent. fine, and from 1lb Troy of this metal are coined 66 shillings. The pound sterling is said to be equal to £1 4s. 4d. currency or \$4.86$\frac{2}{3}$, the pound currency being \$4. In Martin and Trübner's "Currency," the Canadian 10-cent piece is said to weigh 38·42 grs. and to be $\frac{9}{10}$ fine, but an analysis by Professor Croft shows that the fineness is that of sterling. The American mint asserts the value of this piece to be about $9\frac{3}{8}$ cents, their dollar containing 345·6 grs. pure silver.

Examine the consistency of these stateme ts.

II.—ALGEBRA.

1. Multiply together $x^2-\frac{9}{4}ax+\frac{1}{2}a^2+\frac{3}{4}x-\frac{3}{4}a+\frac{1}{4}$ and $x^2+\frac{13}{4}ax-a^2-\frac{1}{4}x+\frac{3}{2}a-\frac{1}{2}$.

Divide the product by $\frac{1}{4}x^2+\frac{1}{2}ax-2a^2-\frac{1}{4}x+2a-\frac{1}{2}$ and extract the square root of the quotient.—*Toronto University, 1865.*

NOTE.—1st expression $= \frac{1}{4}(4x-a+1)(x-2a+1)$. See *Teachers' Handbook*, p. 72.

2nd expression $= \frac{1}{4}(4x-a+1)(x-4a-2)$

Divisor $= \frac{1}{4}(x-2a+1)(x+4a-2)$

$\therefore$ Quotient $= \frac{1}{4}(4x-a+1)^2$, and sq. rt. $= 2x-\frac{1}{2}a+\frac{1}{2}$.

2. Prove $(a^2+b^2+c^2)^3+2(ab+bc+ca)^3-3(b^2+a^2+c^2)(bc+ca+ab)^2$
$=(a^3+b^3+c^3-3abc)^2$.

—*Cambridge, 1862.*

NOTE. Put $a^2+b^2+c^2=x$; $ab+bc+ca=y$; and observe that $x+2y=(a+b+c)^2$. Also put $a+b+c=z$, and observe that $a^3+b^3+c^3-3abc=z(x-y)$. Left hand member becomes $x^3+2y^3-3xy^2$, *i.e.*, $(x-y)^2(x+2y)$; *i.e.*, $z^2(x-y)^2$ *Q. E. D.*

3. If $z=\sqrt{(ay^2-a^2)}\div y$, and $y=\sqrt{(ax^2-a^2)}\div x$, prove that $x=\sqrt{(az^2-a^2)}\div z$.

—*Toronto University, 1869.*

NOTE.—$y^2z^2=ay^2-a^2$ $\therefore$ $x^2y^2z^2=ax^2y^2-a^2x^2$ (A)
$x^2y^2=ax^2-a^2$. Substitute for x^2y^2 in (A) and thus eliminate y.

4. Prove

$$\frac{y-z}{1+yz}+\frac{z-x}{1+zx}+\frac{x-y}{1+xy}=\frac{(y-z)(z-x)(x-y)}{(1+yz)(1+zx)(1+xy)}.$$

—*Toronto, 1871.*

NOTE.—Put the left hand member $=V$, clear of fractions, and we have,

$(y-z)(1+zx)(1+xy)+(z-x)(1+yz)(1+xy)+(x-y)(1+yz)(1+zx)$ $=V(1+yz)(1+zx)(1+xy)$ Factor left hand member by putting $x=y$, &c. (see *Teachers' Handbook*, p. 85), and $(y-z)(z-x)(x-z)=V(1+yz)(1+zx)(1+xy)$. Divide through and $V=$ &c.

5. If $(ay+bx)\div c=(cx+az)\div b=(bz+cy)\div a$, then will

$$\frac{x}{a}\div b^2+c^2-a^2=\frac{y}{b}\div(c^2+a^2-b^2)=\frac{z}{c}\div(a^2+b^2-c^2).$$

—*Toronto University, 1872.*

NOTE.—Put each of the given factors $=m$, whence
$acy+bcx=c^2m$, $bcx+abz=b^2m$, $abz+acy=a^2m$,
and by addition $(acy+bcx+abz)\div abc(a^2+b^2+c^2)=\frac{m}{2bc}=$ any one of required relations. Again combining,
$m(b^2+c^2-a^2)=2bcx$. $\therefore \frac{x}{a}\div(b^2+c^2-a^2)=\frac{m}{2abc}$, nd similarly for the other two.

6. If $x\div a(y-z)=y\div b(z-x)=z\div c(x-y)=1$, prove that $ab+bc+ca=-1$.

—*Toronto University, 1882.*

NOTE.—Clear of fractions and transpose, and
$x-ay+az=bx+y-bz=-cx+cy+z=0$.
Eliminate x from (1) and (2), also from (2) and (3) and
$\frac{y}{z}=\frac{(a+1)b}{ab+1}=\frac{(c-1)b}{(b+1c)}$, from which $ab+bc+ca=-1$.

7. If $x^2+2ayz=y^2+z^2$, $y^2+2bzx=z^2+x^2$, $z^2+2cxy=x^2+y^2$ show that $x(a+bc)=y(b+ac)=z(c+ab)$, and also that
$(1-a^2)(a+bc)^2=$ anl $=$ anl.

—*Toronto University, 1870.*

NOTE.—Transpose so that
$x^2=y^2+z^2-2ayz=y^2-z^2+2bzx=z^2-y^2+2cxy$.
Taking (1)−(2), (1)−(3), and (2)+(3) we get
$z-ay-bx=0$, $y-az-cx=0$, $x-cy-bz=0$; whence
$x=cy+bc=(y-az)\div c=(z-ay)\div b$.
From the latter pair $y(b+ac)=z(c+ab)$, and by symmetry $=x(a+bc)$, which is the first part.
Resume $z-ay-bx=0$, &c. Eliminate x from (1) and (2), from (1) and (3), and (2) and (2), and we get

$$\frac{y}{z}=\frac{c+ab}{b+ac}=\frac{1-b^2}{a-bc}=\frac{a-bc}{1-c^2}.$$

Now the square of the first is equal the product of the other two equal fractions,
$\therefore (1-c^2)(c+ab)^2=(1-b^2)(b+ac)^2$,
whence by symmetry, &c.

8. If a, b, c be the roots of $x^3+px^2+qx+r=0$, show that $\frac{a+b}{c}$, $\frac{b+c}{a}$, $\frac{a+c}{b}$ are the roots of the equation

$$rx^3+(3r-pq)x^2+(r^3-2pq+r)x+(r-pq)=0.$$

—*Toronto University, 1872.*