APPENDIX II.

THEORY OF THE MECHANICAL INTEGRATOR.

As we proceed from B to C by way of A (see figure), x changes from OD to OE and $\int ydx$ is the area $DBACE$; but, if we proceed from C to B by way of P_1, each element of area such as ydx is negative since dx is negative, and hence $\int ydx$ is the area $CEDB$, but is negative. Hence, if we sum the elements such as ydx in the order of proceeding clockwise round the curve, the result $= DBACE - BDEC = BAPC$, the area of the curve. Let A = this area, M = sum of the moments of the elements of the area with respect to OX, I = moment of inertia of the area, also with respect to OX. Then

$$A = \int ydx,$$

$$M = \int ydx \cdot \frac{y}{2} = \frac{1}{2}\int y^2\,dx,$$

$$I = \int ydx \cdot \frac{y^2}{3} = \frac{1}{3}\int y^3\,dx.$$

In the integrator one end of a sweeping bar traces a closed curve while the other end is constrained to describe a straight line OX. This part of the instrument is therefore a planimeter, and the area of the curve $= bc_1n_1$, where b is the length of the sweeping bar, c_1 the circumference of the wheel W_1, which the bar carries, and n_1 the change of reading of this wheel when the circuit of the curve has been made. The end of the bar which describes the