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Pressure Rise Caused by Gradual Gate Closure

Application of Professor Joukovsky's Theory of Maximum Water-Hammer—Solution of Problem by Arithmetic Integration—Derivation of Formulas—Paper Presented to the American Society of Civil Engineers

By NORMAN R. GIBSON

Hydraulic Engineer, Hydraulic Power Co., Niagara Falls

THE purpose of this paper is to show how the excess pressure in penstocks caused by the gradual closing of turbine gates may be determined from Professor Joukovsky's theory of water-hammer. It will be assumed that the theory of pressure waves, their amplitudes, and speeds of propagation, as formulated by him and proved by his experiments, may be accepted as correct.

At the risk of wearying the reader who is familiar with Joukovskys' work, it is necessary, in connection with what is to follow, to summarize as briefly as possible the principles demonstrated a number of years ago by the distinguished Russian. For a partial translation of his work, the reader is referred to Miss O. Simin's paper entitled "Water Hammer," in the Proceedings of the American Water Works Association, 1904, page 341, which should be examined carefully by every student of this subject.

Joukovsky's experiments, made in 1908 at Moscow, were confined to the instantaneous stopping of the flow of water in long pipes. By his experiments he was able to prove the soundness of his analytical determination of the maximum rise of pressure that would occur when the flow of water in a pipe was suddenly arrested. The casual thinker at first would imagine that—as force is equal to the product of mass by acceleration—an infinite pressure would be produced in a pipe if the water flowing in it were stopped instantaneously. On second thought, he would realize that neither the water column nor the walls of the pipe are rigid, and therefore the pressure caused by the shock of stopping the flow suddenly is relieved by the slight compression of the water and the expansion of the walls of the pipe. It was the effect of these two factors that was determined by Professor Joukovsky. He showed that the shock pressure is transmitted along the column of water in the pipe in waves similar to sound waves; and that the shock pressure is proportional to the destroyed velocity of flow and to the speed of propagation of the pressure waves. This speed depends on the compressibility of water, on the elasticity of the materials of the pipe, and on the ratio of thickness of the walls of the pipe to its diameter. In other words, if the speed of the pressure wave is known, the maximum pressure produced (called water-hammer) by instantaneously stopping water flowing in a pipe at any velocity may be calculated.

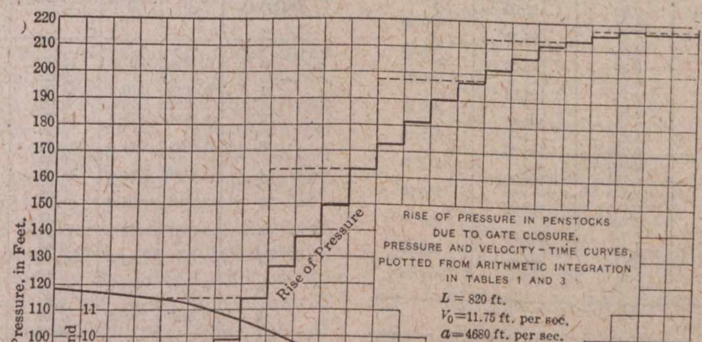

FIG. 1—PRESSURE AND VELOCITY-TIME CURVES PLOTTED FROM TABLES 1 AND 3

Joukovsky's formula for water-hammer is:—

$$h = aV/g \quad \text{.......} (1)$$

where h = excess pressure, in feet;
V = velocity of flow in the pipe, in feet per second;
g = gravitational unit, in feet per second per second;
and a = velocity of the pressure wave, in feet per second, which is determined by the formula:—

$$a = 12 \div [(W/g)(1/k + d/Ee)]^{1/2} \quad \text{.......} (2)$$

where W = weight of a cubic foot of water, in pounds;
k = voluminal modulus of water, in pounds per square inch;
d = diameter of pipe, in inches;
e = thickness of pipe walls, in inches;
E = modulus of elasticity of material of pipe walls, in pounds per square inch.

Joukovsky showed also that the shock pressure is transmitted along the pipe with constant intensity and "at constant velocity, which seems to be independent of the intensity of the shock."

"The speed of propagation of the pressure wave remains the same, whether the shock is caused by arresting the flow of a column of water moving in a pipe, or by suddenly changing the pressure in the column of water (flowing or standing) in any part and by any other means."

"If the water column continues flowing, such flow exerts no noticeable influence upon the shock pressure. In a pipe from which water is flowing, the pressure wave is reflected from the open end of the pipe, in the same way as from a reservoir with constant pressure."

"The phenomenon of periodical vibration of the shock pressure is completely explained by the reflection of the pressure wave from the ends of the pipe, i.e., from the gate and from the origin [of the pipe]."

(These quotations are from Miss Simin's "Water Hammer.")

Pressure waves, after travelling up the penstock to the