The Quadrature of a Circle Determined by its Superficies.

magnitude of a circle can be found by any criterion, independent of the circumference, the circumference will also be given.



With radius equal to 1 describe the circle A B C D, and draw the diameter A C, B D, perpendicular to each other, in the quadrant A B, draw the chord E B, equal to radius, and join O E, then will the arc E $B=60^\circ$, and the sector contained between the two radii (O E, O B,) and the arc E B, equal in magnitude to one-sixth part of the circle A B C D, from the centre O let fall upon the chord E B, the perpendicular O M, the chord (E B) is bisected in M, produce O M to L, the arc (E B), is bisected in L, draw E L, L B, and each will be the chord of half the arc, EB, and the lines E M, M B, (each equal to half the radius) will each be equal to sine of half the arc E B, and the lines O M, M L, equal respectively to the co-sine and versed sine of half the arc E B, it also is obvious by inspection, that the versed sine (M L) is equal to radius (OL) minus the co-sine (O M). These things being premised let R, S, Cos, VS, represent the Radius, Sine, Co-sine, and Versed Sine respectively, then will the magnitude or superficial contents of the circular sector E O B = R $xS 30^{\circ}$, + $SxVS 15^{\circ}x2$, + $SxVS 7^{\circ} 30'$ x4 + SxVS 30° 45′x8, + SxVS 1° 52′ **30-2x16 + SxVS 56' 15-2x32 + SxVS** 28' 7-2 30-3x64 + SxVS 14' 3-2 45-3 x128 + SxVS 7' 1-2 52-3 30-4x256

+SxVS 3' 30-2 56-3 15-4x512 + SxV S 1' 45-2 28-3 7-4 30-5x1024 + SxVS 52-2 44-3 3 4 45-5x2048, &c.

It will be observed that the above sines may be continued ad inpuiturn, and that no finite number of the terms will be equal to the circular sector EOB, which is literally true; yet when the arc E B, has been subdivided by 12 bisections, the last found arc (being something less than one minute of a degree) will so nearly coincide with its chord, that the cosine of half the arc may be esteemed/ equal to radius, and consequently the versed sine will vanish. But should it be necessary to extend the sines in the present question, the results may be obtained without the tedious process of calculating the sines, &c.-For it will be observed in the process, that the product of each term is but a very small fraction above the product of the term immediately preceding it. In the resolution of the premises (having calculated the sines, co-sines, and versed sines upon trigonometrical principles to twelve places of decimals, and extended the for egoing sines as far as would effect any of the figures within these limits) I have found the magnitude or superficies of the circular sector (E O B) to be equal numerically to 523598733797, which number multiplied by 6, gives the magnitude of the circle A B C D=3.141592402782, the radius by hypothesis being = 1. if the diameter is taken = 1. the circumference will be = 3.141592402782. Dr. Hutton in his treatise on mensuration, gives the diameter to the circumference in the ratio of 1. to 3.141592653589, making the circumference greater than the above calculation .000000250807, but by your correspondent's theory the diameter to the circumference is found to be in the ratio of 1: 3.142798314380, making the circumference of a circle (whose diameter is=1.) greater than is usually supposed in the amount of .0011982976, and although this (abstractedly considered) is but a small fraction, yet in a circle equal to the

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