

process than that of

putting $x = r + h$,
of h .

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former problem.

the expression

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32.

compute the
5.

ssive sums ob-
corresponding
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stopping yet
efficient of h^2 .

he first term.
he coefficient

ed as follows:

+4
28
32

8,

powers of h .

Coefficients,	2	-7	+5	-2	+6	-8
Products by 3,		6	-3	+6	+12	+54
First sums,		-1	+2	+4	+18	+46
Second products,		+6	+15	+51	+165	
Second sums,		+5	+17	+55	183	
Third products,		6	33	150		
Third sums,		11	50	205		
		6	51			
		17	101			
		6				
		23				

$$\text{Result, } F(3+h) = 2h^5 + 23h^4 + 101h^3 + 205h^2 + 183h + 46.$$

EXERCISES.

1. Compute $2h^5 + 23h^4 + 101h^3 + 205h^2 + 183h + 46$, when $h = x - 3$.

2. Compute $x^2 - 1x + 7$ for $x = -4 + h, -3 + h$, etc., to $+3 + h$.

Proof of the Preceding Process. If we develop the expression

$$a(h+r)^n + b(h+r)^{n-1} + c(h+r)^{n-2} + d(h+r)^{n-3} + \text{etc.},$$

and collect the coefficients of like powers of h , we shall find

$$\text{Coef. of } h^n = a,$$

$$h^{n-1} = nar + b,$$

$$h^{n-2} = \binom{n}{2} ar^2 + (n-1)br + c, \quad (A)$$

$$h^{n-3} = \binom{n}{3} ar^3 + \binom{n-1}{2} br^2 + (n-2)cr + d,$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$h^{n-s} = \binom{n}{s} ar^s + \binom{n-1}{s-1} br^{s-1} + \binom{n-2}{s-2} cr^{s-2} + \text{etc.}$$

Now examining Ex. 2 preceding, it will be seen that we can make the computation by columns, first computing the whole left-hand column and thus obtaining the coefficient of h^{n-1} , then computing the next column, thus obtaining the coefficient of h^{n-2} , and so on. Commencing in this way, and using the literal coefficients, a, b, c , etc., and the literal factor r , we shall have the results: