B2.0 DERIVATION OF $\frac{\partial(\Delta V_a)}{\partial a}$ FOR CIRCULAR ORBITS

Hohmann transfer delta-V is given by

$$\Delta V_a = \sqrt{\frac{\mu}{a+\Delta a}} - \sqrt{\frac{\mu}{a}} + \sqrt{\frac{\mu}{a+\frac{\Delta a}{2}}}\left(\sqrt{\frac{a+\Delta a}{a}} - \sqrt{\frac{a}{a+\Delta a}}\right) \qquad \text{(B2.1)}$$

Taking Taylor's expansion of $\frac{1}{\sqrt{a+\Delta a}}$ and $\sqrt{a+\Delta a}$

$$\frac{1}{\sqrt{a+\Delta a}} \doteq \frac{1}{\sqrt{a}} - \frac{\Delta a}{2\sqrt{a^3}} \qquad \text{(B2.2)}$$

$$\sqrt{a+\Delta a} \doteq \sqrt{a} + \frac{\Delta a}{2\sqrt{a}} \qquad \text{(B2.3)}$$

Substituting Equations (B.2) and (B.3) into (B.1) gives

$$\frac{\Delta V_a}{\sqrt{\mu}} \doteq \frac{1}{\sqrt{a}} - \frac{\Delta a}{2\sqrt{a^3}} - \frac{1}{\sqrt{a}} + \left(\frac{1}{\sqrt{a}} - \frac{\Delta a}{4\sqrt{a^3}}\right)\left(\frac{\sqrt{a} + \frac{\Delta a}{2\sqrt{a}}}{\sqrt{a}} - \sqrt{a}\left(\frac{1}{\sqrt{a}} - \frac{\Delta a}{2\sqrt{a^3}}\right)\right)$$

Simplifying and setting $\Delta a^2 = 0$ gives

$$\frac{\Delta(\Delta V_a)}{\Delta a} \doteq \sqrt{\frac{\mu}{4a^3}}$$

and, as $\Delta a \rightarrow 0$

$$\boxed{\frac{\partial(\Delta V_a)}{\partial a} = \frac{1}{2a}\sqrt{\frac{\mu}{a}}}$$