(*b*) $\tan\frac{1}{2}(\alpha+\beta)-\tan\frac{1}{2}(\alpha-\beta)=\dfrac{\sin\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\alpha+\beta)}-\dfrac{\sin\frac{1}{2}(\alpha-\beta)}{\cos\frac{1}{2}(\alpha-\beta)}$.

$$=\frac{\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)-\cos\frac{1}{2}(\alpha+\beta)\sin\frac{1}{2}(\alpha-\beta)}{\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)}$$

$$=\frac{\left\{\sin\frac{1}{2}(\alpha+\beta)-\frac{1}{2}(\alpha+\beta)\right\}}{\frac{1}{2}\left\{\cos\left(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right)\right\}}=\frac{2\sin\beta}{\cos\alpha+\cos\beta}.$$

(*c*) $\sin\frac{\pi}{15}=\sin\frac{14\pi}{15}=2\sin\frac{7\pi}{15}\cos\frac{7\pi}{15}=2\sin\frac{8\pi}{15}\cos\frac{7\pi}{15}$

$=2^2\sin\frac{4\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}=2^3\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}$

$=2^4\sin\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}$

$\therefore\ \frac{1}{2^4}=\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}$ (1)

Again

$\sin\frac{3\pi}{15}=\sin\frac{12\pi}{15}=2\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}=2^2\sin\frac{3\pi}{15}\cos\frac{6\pi}{15}\cos\frac{3\pi}{15}.$

$\therefore\ \frac{1}{2^2}=\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}$ (2)

and $\frac{1}{2}=\cos\frac{5\pi}{15}$ or $\cos 60^0$ (3)

$\therefore$ multiplying (1), (2) and (3)

$\frac{1}{2^7}=\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

6. Show that the following relations hold good for every triangle :

(*a*) $c=a\cos B+b\cos A$.

(*b*) $c^2=a^2+b^2-2ab\cos C$.

(*c*) $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$.

7. In a triangle $\tan\frac{A}{2}=\frac{5}{6}$ and $\tan\frac{B}{2}=\frac{20}{37}$. Find $\tan\frac{C}{2}$ and prove that $a+c=2b$.

(*a*) $\cot\frac{C}{2}=\tan\frac{A+B}{2}=\dfrac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\dfrac{\frac{5}{6}+\frac{20}{37}}{1-\frac{5}{6}\cdot\frac{20}{37}}=\frac{5}{2}$

$\therefore\ \tan\frac{C}{2}=\frac{2}{5}$

(*b*) $\tan\frac{A}{2}\tan\frac{C}{2}=\frac{5}{6}\times\frac{2}{5}=\frac{1}{3}$

$\therefore\ \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}\ \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}=\frac{1}{3}$

or $\frac{s-b}{s}=\frac{1}{3}$ $\quad 2s=3b \quad a+b+c=3b \quad \therefore\ a+c=2b$

8. Define a logarithm and prove :

(*a*) $\log_a mn=\log_a m+\log_a n$.

(*b*) $\log_a m^n=n\log_a m$.