7. Put n successively equal 1, 2, 3, &c., and we have the sums of 1, 2, 3, &c., terms=2, 6, 12, 20, 30, 42, &c., respectively.

Hence the series must be

$$2+6+12+20+\&c. =2\{1+3+6+10+15+\&c.+\tfrac{1}{2}n(n+1)\}$$

Hence required sum

$$=2(2+3+4+\&c.\dots+n).\quad \text{Put}$$

$$s+1=1+2+3+4+\&c.+n=\frac{n}{2}(n+1).$$

$\therefore$ Required sum

$$=\left\{\frac{n}{2}(n+1)-1\right\}2=n^2+n-2=(n-1)(n+2).$$

8. Book-work. If we insert n Geometric means between a and b, we have $r=\left(\frac{a}{b}\right)^{\frac{1}{n+1}}$ $\therefore$ in this case we get

$$r_1=\left(\frac{x^2}{x}\right)^{\frac{1}{n+1}},\quad r_2=\left(\frac{x^3}{x}\right)^{\frac{1}{n+1}},\quad r_3=\left(\frac{x^4}{x}\right)^{\frac{1}{n+1}},\ \&c.,\ r_n=\left(\frac{x^{n+1}}{x}\right)^{\frac{1}{n+1}}$$

i.e. $r_1 = x^{\frac{1}{n+1}}$, $r_2 = x^{\frac{2}{n+1}}$, $r_3 = x^{\frac{3}{n+1}}$, &c. $r^n = x^{\frac{n}{n+1}}$.

$$\therefore \frac{r_2}{r_1}+\frac{r_3}{r_2}+\&c.=x^{\frac{1}{n+1}}+x^{\frac{1}{n+1}}+\ \&c.\ x^{\frac{1}{n+1}}=n\ x^{\frac{1}{n-1}}.$$

9. $Cr=C_{r-1}\times\frac{n-r+1}{r}$, $\therefore Cr > C_{r-1}$, so long as $\frac{n-r+1}{r}>1$, or $n+1>2r$, and the greatest value of r is the integer next below $\frac{1}{2}(n+1)$. When n is even $r=\frac{1}{2}n$, when n is odd $r=\frac{1}{2}(n-1)$.

No. of combinations of 13 things is greatest 6 together. Hence take 6 out of the 13 group and 2 out of the 8 group. Remove A and B, and take 5 out of the 12 group and 1 out of the 7 group This gives 792 groups of 5 and 7 groups of 1, and each of the former may be combined with each of the latter to form a group of 6, *i.e.* there are 792 × 7=5544 groups of 6, to which if we now add A and B, they meet 5544 times. The total number of selections possible is 1716 × 28.

10. Book-work.

$$\frac{1}{\sqrt{3}}=3^{-\frac{1}{2}}=(4-1)^{-\frac{1}{2}}=\left\{4\left(1-\frac{1}{2^2}\right)\right\}^{-\frac{1}{2}}=\frac{1}{2}\left(1-\frac{1}{2^2}\right)^{-\frac{1}{2}}$$

$$\text{Now } (1-x)^{-\frac{p}{q}}=1+\frac{p}{q}x+\frac{p(p+q)}{\underline{|2}\,.q^2}x^2+\frac{p(p+q)(p+2q)}{\underline{|3}\,.q^3}x^3+\&c.$$

$$\therefore \frac{1}{2}\left(1-\frac{1}{2^2}\right)^{-\frac{1}{2}}=\frac{1}{2}\left\{1-\frac{1}{2}\cdot\frac{1}{2^2}+\frac{1.3}{\underline{|2}\,.2^2}\cdot\frac{1}{2^4}+\frac{1.3.5}{\underline{|3}\,.2^3}\cdot\frac{1}{2^6}+\&c.\right\}$$

which is the given series.

$$(1+x)^n=\left(1-\frac{x}{1+x}\right)^{-n}.\ \text{Now coeff. of } x^r \text{ on left hand}$$

$$=\frac{n(n-1)\dots(n-r+1)}{\underline{|r}}.\ \text{Expansion of right hand}$$

$$=1+\frac{n}{1}\cdot\frac{x}{1+x}+\frac{n(n+1)}{\underline{|2}}\cdot\frac{x^2}{(1+x)^2}+\&c.$$

$$+\frac{n(n+1)(n+2)\dots(n+r-1)}{\underline{|r}}\cdot\frac{x^r}{(1+x)^r},$$

and the succeeding terms will contain only powers or r higher than x^r. We see that

$$\frac{x^r}{(1+x)^r}=x^r\left(1+x\right)^{-r}=x^r\left(1-\frac{r}{1}.x+\&c.\right)=x^r-rx^{r+1}+\&c.$$

$$\frac{x^{r-1}}{(1+x)^r}=x^{r-1}\left(1+x\right)^{-(r-1)}=x^{r-1}\left\{1-\frac{r-1}{1}x+\&c.\right\}$$

$$=x^{r-1}-\frac{r-1}{1}x^r+\&c.\quad \frac{x^{r-2}}{(1+x)^{r-2}}=x^{r-2}.\left(1+x\right)^{-(r-2)}$$

$$=x^{r-2}\left\{1-\frac{r-2}{1}.\ x+\frac{(r-2)(r-1)}{\underline{|2}}x^2+\&c\right\}$$

$$=x^{r-2}-\frac{r-2}{1}x^{r-1}+\frac{(r-2)(r-1)}{\underline{|2}}x^r-\&c.$$

Thus each term after the first contains x^r, and the sum of all those coeffs. must equal the coeff. of x^r on the left hand of the identity, *i.e.* we must have

$$\frac{n(n+1)\dots(n+r-1)}{\underline{|r}}(1)+\frac{n(n+1)\dots(n+r-2)}{\underline{|r-1}}-\left(-\frac{r-1}{1}\right)$$

$$+\frac{n(n+1)\dots(n+r-3)}{\underline{|r-2}}.\left(\frac{\overline{r-2}.\ \overline{r-1}}{\underline{|2}}\right)+\frac{n(n+1)\dots(n+r-4)}{\underline{|3}}$$

$$\left(-\frac{\overline{r-3}.\ \overline{r-2}.\ \overline{r-1}}{\underline{|3}}\right)+\&c.-(-1)^r$$

$$\frac{n(n-1)\dots(n-r+1)}{\underline{|r}}.\ \text{And this is the required relation.}$$

MANITOBA TEACHERS' EXAMINATIONS—*1882*.

MENSURATION.—1st & 2nd Classes.

Examiner—Stewart Mulvey. *Time—Two Hours.*

A. Give the rules for finding:
  (*a*) The area of an equilateral triangle.
  (*b*) A Trapezium.
  (*c*) A Circle.
  (*d*) A Sector of a circle.
  (*e*) A Segment of a circle.

B. Give the rules for finding:
  (*a*) The solidity of any pyramid.
  (*b*) The contents of a frustrum of a pyramid.
  (*c*) The cubical contents of any prism.
  (*d*) The solidity of a wedge.

1. The sides of a triangle are respectively 13, 14, and 15 feet. What is its area and the perpendicular on the *greatest side?*

2. One of the sides of an isosceles triangle is 7 feet and the base is 12 feet. What is its area?

3. Required the area of a segment of which the height is 20, the diameter of the circle being 23?

4. The chord of an arc less than a semi-circle is 336, and the diameter is 625. Required the length of the arc?

5. The longer axes of a prolate spheroid is 55 and the shorter 33 inches What is the solid contents of the spheroid?

6. . nd the difference between the area of a triangle whose sides are 6, 8, and 10 feet, and the area of an equilateral triangle having an equal perimeter?

7. How many standard or imperial gallons in a cistern of the following dimensions, viz.: Bottom diameter, 60 inches; middle diameter, 50 inches; and top diameter, 65 inches. Depth of cistern, 40 inches?

8. How many bushels of barley and imperial gallons would the above cistern contain when filled 30 inches from the bottom?

Solutions.

A. (*a*) Area=·433(side)^2.

(*b*) If a, b, c, d, be the four sides, area=$\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where $s=\frac{1}{2}(a+b+c+d)$. *If d be one of the diagonals and* p and p_1 the perps. on it from the opp. angles, area$=\frac{1}{2}d(p+p_1)$.

(*c*) Area=πr^2, where π=3·14159+, and r=radius.

(*d*) If l=length of arc, area=$\frac{1}{2}lr$.

(*e*) Area of segment=area of sector—area of triangle whose base is chord of arc and vertex at centre.

B. (*a*) Solidity=$\frac{1}{3}$(area of base × perp. height).

(*b*) Solidity=$\frac{1}{3}$(area of base+area of top+mean proportional between then) .: perp. height.

(*c*) Solidity=area of base × perp. height.

(*d*) Solidity=$\frac{1}{6}$(twice length of base+length of edge) × breadth of base × perp. height.

1. $s=\frac{1}{2}(13+14+15)=21$; $s-a$, $s-b$, $s-c$, =resp. 8, 7, and 6.

$\therefore$ area= $\sqrt{21\times8\times7\times6}=\sqrt{7\times3\times4\times2\times7\times3\times2}$

$=\sqrt{7^2\times3^2\times4^2}=7.3.4=84.$

Also 2 area=15 × perp.=2 × 84, $\therefore$ perp.=2 × 84 ÷ 15=11$\frac{1}{5}$.

2. 2 area=12 × perp. But perp.$^2=7^2-6^2=13$, $\therefore$ perp.$=\sqrt{13}$.

$\therefore$ area $=6\sqrt{13}=6\times3{\cdot}65.=21{\cdot}9+$