SCHOOL WORK.

MATHEMATICS.

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SOLUTIONS TO PROBLEMS IN FEBRUARY NUMBER.

73. Solve $x^{y+z}=y^{x+z}=z^{x+y}$ and $x^a=y^b=z^c$ where b is the harmonic mean between a and c.

73. Take logarithms, $\therefore$ $y+z)$ log. $x=(z+x)$ log. $y=$ & & a log. $x=b$ log. $y=c$ log. z & $b=\dfrac{2ac}{a+b}$ $\therefore$ by division $\dfrac{y+z}{a}=\dfrac{x+y}{c}$, but $y=x^{\frac{a}{b}}$ & $z=x^{\frac{a}{c}}$ $\therefore$ $c\left(x^{\frac{a}{b}}+x^{\frac{a}{c}}\right)=a\left(x+x^{\frac{a}{b}}\right)$ $\therefore$ $x=0$ &c., is one set of values Also $c\left(x^{\frac{a-b}{b}}+x^{\frac{a-c}{c}}\right)=a\left(1+x^{\frac{a-b}{b}}\right)$ Substitute for b in terms of c and we get $(c-a)\,x^{\frac{a-c}{2c}}+c\,x^{\frac{a-c}{c}}=a.$

Solving this quadratic $x=\left(\dfrac{a}{c}\right)^{\frac{2c}{a-c}}$ & $y=\left(\dfrac{a}{c}\right)^{\frac{a+c}{a-c}}$ & $z=\left(\dfrac{a}{c}\right)^{\frac{2a}{a-c}}$. Other values may be found.

74. If $\dfrac{a^2-b^2}{l-m}=\dfrac{ab}{c}$ and $\dfrac{b^2-c^2}{m-n}=\dfrac{bc}{a}$ prove that $\dfrac{c^2-a^2}{n-l}=\dfrac{ca}{b}$.

74. $\dfrac{a^2-b^2}{l-m}=\dfrac{ab}{c}$ $\therefore$ $c\,\dfrac{(a^2-b^2)}{ab}=l-m,$

$\therefore$ $l-m=\dfrac{ac}{b}-\dfrac{bc}{a}$ also $m-n=\dfrac{ab}{c}-\dfrac{ac}{b}$

Adding we have $n-l=\dfrac{bc}{a}-\dfrac{ab}{c}$

$$\therefore\ \frac{c^2-a^2}{n-l}=\frac{ca}{b}.$$

75. Three equal circles of radii r touch each other (two and two); find the area of the space intercepted between the circles, and show that the radii of the circles that touch all three are $\dfrac{2\pm\sqrt{3}}{\sqrt{3}}\,r$.

75. (a) By joining the centres we have an equilateral triangle of side $2\,r$, whose area is $r^2\sqrt{3}$. The area of each sector thus formed is $\frac{1}{6}\,\pi\,r^2$. Subtracting the areas of the three sectors from the triangle we have $r^2\sqrt{3}-\dfrac{\pi\,r^2}{2}$.

(b) The intersection of the perpendiculars on the opposite sides in the above triangle is the centre of both circles required and the radii are $\frac{2}{3}\cdot r\sqrt{3}\pm r$.

76. The hour, minute and second hands being on the same centre and moving uniformly; find in what time the second hand would divide the angle between the hour and minute hands in the ratio of $m:n$ after a minutes past b o'clock.

76. In $\dfrac{60\,(60\,na+5\,nb+12\,mb)}{708\,m+719\,n}$ seconds.

77. The angles of a triangle ABC are bisected by lines cutting the sides; show that the product of the alternative segments of the sides $=\dfrac{a^2b^2c^2}{(a+b)(b+c)(c+a)}$.

77. By Euc. VI. 3, the side AB (c) is cut in the ratio of $a:b$, $\therefore$ the segments are $\dfrac{bc}{a+b}$ & $\dfrac{ac}{a+b}$ similarly for the other sides. Hence the product of the alternate segments is easily found.

78. From a point within a circle straight lines are drawn, such that the circumference divides them in a given ratio; find the locus of the external (or internal) points.

78. Let A the given point; C the centre of the circle. Produce AC to D so that AC and CD are in the given ratio. By Euc. VI. 2 and 4, it can easily be shown that the distance from D to the extremeties of all the lines is constant. Hence the locus is a circle with centre D.

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