

Therefore, from (7), keeping in view that $A = -\frac{47}{500}$,

$$h = \frac{47}{8} \left(\frac{9439}{13 \times 25 \times 125} \right)^2.$$

Also, from the first two of equations (8), $B = \frac{39}{100}$, and $B' = \frac{91 \times 9439}{500 \times 4225}$

$$\therefore B'\sqrt{z} = \frac{91\sqrt{5}}{500},$$

and

$$hz + h\sqrt{z} = \frac{1}{625} \left\{ \frac{47}{8} (21125 + 9439\sqrt{5}) \right\}$$

Therefore,

$$\left. \begin{aligned} u_1^5 &= \frac{13}{100} \left(3 + \frac{7\sqrt{5}}{5} \right) + \frac{1}{625} \sqrt{\left\{ \frac{47}{8} (21125 + 9439\sqrt{5}) \right\}} \\ u_4^5 &= \frac{13}{100} \left(3 + \frac{7\sqrt{5}}{5} \right) - \frac{1}{625} \sqrt{\left\{ \frac{47}{8} (21125 + 9439\sqrt{5}) \right\}} \\ v_2^5 &= \frac{13}{100} \left(3 - \frac{7\sqrt{5}}{5} \right) + \frac{1}{625} \sqrt{\left\{ \frac{47}{8} (21125 - 9439\sqrt{5}) \right\}} \\ u_3^5 &= \frac{13}{100} \left(3 - \frac{7\sqrt{5}}{5} \right) - \frac{1}{625} \sqrt{\left\{ \frac{47}{8} (21125 - 9439\sqrt{5}) \right\}} \end{aligned} \right\} \quad (31)$$

Therefore the root of the given quintic is known.

§13. To verify this result, we have $9439\sqrt{5} = 21106.2456$. Therefore

$$\frac{1}{625} \sqrt{\left\{ \frac{47}{8} (21225 + 9439\sqrt{5}) \right\}} = .79696796,$$

$$\frac{1}{625} \sqrt{\left\{ \frac{47}{8} (21225 - 9439\sqrt{5}) \right\}} = .01679462.$$

Also,

$$\frac{13}{100} \left(3 + \frac{7\sqrt{5}}{5} \right) = .79696437$$

and

$$-\frac{13}{100} \left(3 - \frac{7\sqrt{5}}{5} \right) = .01696437.$$

Therefore

$$\begin{aligned} u_1^5 &= 1.593932, & u_1 &= 1.09773 \\ -u_4^5 &= .00000359, & -u_4 &= .08147 \\ -u_2^5 &= .00016975, & -u_2 &= .17618 \\ -u_3^5 &= .03375899, & -u_3 &= .50778. \end{aligned}$$

Therefore $u_1 + u_2 + u_3 + u_4 = .3323$. Therefore

$$x^5 = .004052$$

$$3x^3 = .331248$$

$$2x = .66448$$

$$.99978$$