It may be observed, that the two parallelograms exhibited in fig. 2 partially lie on one another, and that the triangle whose base is BC is a common part of them, but that the triangle whose base is DE is entirely without both the parallelograms. After having proved the triangle ABE equal to the triangle DCF, if we take from these equals (fig. 2.) the triangle whose base is DE, and to each of the remainders add the triangle whose base is BC, then the parallelogram $ABCD$ is equal to the parallelogram $EBCF$. In fig. 3, the equality of the parallelograms $ABCD$, $EBCF$, is shewn by adding the figure $EBCD$ to each of the triangles ABE, DCF.

In this proposition, the word *equal* assumes a new meaning, and is no longer restricted to mean coincidence in all the parts of two figures.

Prop. XXXVIII. In this proposition, it is to be understood that the bases of the two triangles are in the same straight line. If in the diagram the point E coincide with C, and D with A, then the angle of one triangle is supplemental to the other. Hence the following property:—If two triangles have two sides of the one respectively equal to two sides of the other, and the contained angles supplemental, the two triangles are equal.

A distinction ought to be made between *equal* triangles and *equivalent* triangles, the former including those whose sides and angles mutually coincide, the latter those whose areas only are equivalent.

Prop. XXXIX. If the vertices of all the equal triangles which can be described upon the same base, or upon the equal bases as in Prop. 40, be joined, the line thus formed will be a straight line, and is called the locus of the vertices of equal triangles upon the same base, or upon equal bases.

A locus in plane Geometry is a straight line or a plane curve, every point of which and none else satisfies a certain condition. With the exception of the straight line and the circle, the two most simple loci; all other loci, perhaps including also the Conic Sections, may be more readily and effectually investigated algebraically by means of their rectangular or polar equations.

Prop. XLI. The converse of this proposition is not proved by Euclid: viz. If a parallelogram is double of a triangle, and they have the same base, or equal bases upon the same straight line, and towards the same parts, they shall be between the same parallels. Also, it may easily be shewn that if two equal triangles are between the same parallels; they are either upon the same base, or upon equal bases.

Prop. XLIV. A parallelogram described on a straight line is said to be *applied* to that line.

Prop. XLV. The problem is solved only for a rectilineal figure of four sides. If the given rectilineal figure have more than four sides, it may be divided into triangles by drawing straight lines from any angle of the figure to the opposite angles, and then a parallelogram equal to the third triangle can be applied to LM, and having an angle equal to E: and so on for all the triangles of which the rectilineal figure is composed.

Prop. XLVI. The square being considered as an equilateral rectangle, its area or surface may be expressed numerically if the number of lineal units in a side of the square be given, as is shewn in the note on Prop. I., Book II.

The student will not fail to remark the analogy which exists between the *area of a square* and *the product of two equal numbers*; and between *the side of a square* and *the square root of a number*. There is, however,