

blackboard demonstration, cannot simplify expressions correctly, is perpetually hampered by what he thinks are trifling mistakes, mistakes primarily due to a want of proper attention to signs when he was first introduced to them, and his infirmity is now rendered difficult to correct on account of that very familiarity with the signs which, in accordance with the proverb prevents their receiving respectful consideration.

One who has understood thoroughly the relations of length, area, and volume, experiences exceptional difficulties when the more complicated notions of mechanics are presented, involving the further conceptions of time and mass. It is a cruel thing to teach a person arithmetic without giving him the advantage of a rational study of square and cubic measures, or to limit his geometrical teaching as to exclude the discussion of similar figures.¹³ A valuable training is missed, and the want of this knowledge seriously impedes, nay, even sometimes entirely prevents further progress. All that has been said of the usefulness of so teaching the early subjects as to prepare the way for the later, may be repeated as causes of difficulty in teaching the later subjects when the way has not been so prepared, and of still greater difficulty if the earlier subjects have been so taught as to obstruct the acquisition of later ideas.

A practical difficulty is sometimes the infrequency and length of lessons; what is wanted for young beginners in mathematics—I suspect that it would be best for all subjects—is frequent short lessons. Some pupils get, say for geometry, two hours a week; now, if this were in four lessons of half-an-hour each, or even six lessons of twenty minutes each, progress would be rapid and the time would

be ample; but when it is in two lessons of one hour each, much of the time is necessarily wasted on account of the incapacity of the pupils for prolonged mental exertion. There should be short lessons with full attention.

Another difficulty of mathematical teaching arises from too great dependence on text-books. The order in which a writer groups the various parts of the subject, is not always that in which the learner can most easily assimilate and digest the matter. Oral teaching overcomes this difficulty which is one of the more advanced rather than of the elementary teaching, for the schoolboy rarely reads a text-book.

I will conclude with a few remarks on *Examinations*. It will probably be argued against me, that the rational teaching I have recommended may be all very well in itself, but it "won't pay" in examination. To those who are inclined to hold this opinion I would say, Make the experiment. After a fair trial, sufficiently long continued, see if boys and girls, whose understanding is kept alive during the whole of their lessons, who are never taught names that do not correspond to ideas in their minds, nor to learn by heart what they don't understand, who are taught to refer to fundamental principles rather than to imitate processes in arithmetic and algebra, who are taught to regard geometry as a chain of reasoning that they are expected to understand and to use for themselves—see if such pupils do not utterly defeat in any well-ordered examination the pupils whose only idea of learning is learning by heart.

Nevertheless, examinations may be so conducted as to oppose the efforts of the teacher. A paper in geometry for instance, which consists of nothing but bald propositions, without the slightest variation in their statement

¹³ London University Matriculation, used as a leaving examination.