Chicago Drainage Canal.

By Rankine's Formula.

$$D = 8{\cdot}025\, c\, A\ \sqrt{h}$$
$$= 8{\cdot}025 \times {\cdot}618\, A\ \sqrt{10}$$
$$= 8{\cdot}025 \times {\cdot}618 \times 3{\cdot}16\, A$$
$$= 15{\cdot}67\, A$$

$$\therefore \quad A = \frac{D}{15{\cdot}67} = \frac{3}{15{\cdot}67}$$
$$= {\cdot}1915 \text{ square feet.}$$

Let x = diameter of orifice in feet.

Then—

$${\cdot}7854\, x^2 = {\cdot}1915$$
$$\therefore x^2 = \frac{{\cdot}1915}{{\cdot}7854} = {\cdot}2439$$
$$x = \sqrt{{\cdot}2439} = {\cdot}494 \text{ feet.}$$
$$= 5{\cdot}84 \text{ inches.}$$

2ndly. Suppose now a second orifice, made in the bottom of the tank, capable of discharging ·2 cubic feet per second at a like head of pressure; what would be the diameter of the orifice ?

Let the same formula and notation be used.

A' = area of second orifice.

Then—

$$A' = \frac{D}{15{\cdot}67} = \frac{{\cdot}2}{15{\cdot}67} = {\cdot}01277 \text{ square feet.}$$

Let x' = diameter of less orifice.

Then—

$${\cdot}7854\, x'^2 = 01277$$
$$\therefore x'^2 = \frac{{\cdot}01277}{{\cdot}7854} = {\cdot}0164$$
$$x' = \sqrt{{\cdot}0164} = {\cdot}128 \text{ feet} = 1{\cdot}54 \text{ inches.}$$

Suppose now, with a constant supply of three cubic feet, both orifices are opened, discharging 3·2 cubic feet per second, the problem is to find how much the level of the tank must fall to establish equilibrium between influx and efflux.

At first blush, one would be led to infer that, with the outflow in excess of the inflow, the tank would eventually be drained of all its contents. But this is not so, as the following computation demonstrates :

Let h_1 = equal the depth of water in the tank or new head of pressure to maintain equilibrium of supply and disdharge.

Then

$$D = 8{\cdot}025\, c\, (A + A_1)\ \sqrt{h_1}$$
$$= 8{\cdot}025 \times {\cdot}618\, ({\cdot}1915 + {\cdot}01277)\ \sqrt{h_1}$$
$$= 8{\cdot}025 \times {\cdot}618 \times {\cdot}2043\ \sqrt{h_1}$$
$$= 1{\cdot}0133\ \sqrt{h_1}$$

Square both sides.

Then

$$D^2 = \overline{1{\cdot}0133)}^2\, h_1$$
$$h_1 = \left(\frac{1{\cdot}0133}{D}\right)^2 = \left(\frac{3}{1{\cdot}0133}\right)^2$$
$$= \overline{2{\cdot}96)}^2 = 8{\cdot}76 \text{ feet.}$$

∴ to maintain equilibrium, there would be a depression in the water surface of the tank of 1·24 feet ; and it would remain at this elevation whiie the conditions of the problem remained unchanged.