THE RELATION OF MATHEMATICS TO ENGINEERING.*

By Prof. A. Ranum.

HOW can we reconcile the fact that many a successful engineer uses very little mathematics in his work with the further well-known fact that the profession of engineering rests to a large extent on a mathematical foundation? This question has many phases, one of which we can answer by pointing out that there is a vast difference between developing the mathematical theory that applies to an engineering problem and merely making use of the theory after it has been developed and put in tabular form by someone else. The latter process does not require very high mathematical attainments, but it is sufficient for many practical purposes. In order to gain more light, however, on this and other similar questions, let us try, if possible, to determine precisely what contributions mathematics has made to engineering; by looking back into the past perhaps we shall discover some general law that will enable us to peer a little into the future.

Engineering has been defined as the art of directing the great sources of power in nature for the use and convenience of man. Now, power implies energy, force, motion. Modern science has shown that all the phenomena of nature, including heat, light, and electricity, are manifestations of energy, modes of motion. In order to direct the forces of nature we must know how they act; we must understand the laws underlying the different kinds of motion, molecular as well as molar. Mechanics is, then, the fundamental science on which engineering depends. The other branches of physics reduce, in the last analysis, to mechanics. Now, in the case of a moving body, molecule, or electron, the first thing we want to know is its velocity, and the next is its acceleration. Both of these are rates of change or derivatives. Hence it is the most natural thing in the world to introduce the calculus into mechanics. The mathematical notion of a derivative is not something imposed upon mechanics from without; it belongs to the very essence of the science. Every waterfall, every bird on the wing, every ray of sunlight, every flash of lightning, when interpreted in mechanical terms, speaks the language of the calculus.

We must guard, however, against the error of supposing that mathematics can furnish us with any of the facts on which the laws governing physical phenomena are based. These facts can only be found by observation and experiment. But when once a precise physical law has been discovered, the function of mathematics is first to provide it with a language adequate to express all its complex and delicate content, and, second, to interpret its hidden meaning and derive the consequences that flow from it, when the other known physical laws are taken into account. This means that the mathematician builds on the given foundation of experimental laws a logical structure, which often contains new theorems of far greater physical significance than the original ones from which they are derived. It is in this sense that mathematics has been described as the master-key that unlocks the secrets of nature. Sometimes, moreover, a mathematical development of this kind leads in the most unexpected fashion to important practical applications. The delicate and exhaustive experiments and far-reaching generalizations of the physicist, the profound and searching analysis and rigorous thinking of the mathematician, the ingenious and practical resourcefulness of the inventor, are all three necessary factors in the progress of engineering. The influence of the last of these, the inventor, although more direct and more easily understood than the others, is not therefore necessarily the most important. On the contrary, his work is often a mere corollary of the scientific research which has prepared the way for him. The history of science furnishes countless illustrations of this. The development of electricity in general, and the discovery of wireless telegraphy in particular, are striking examples which I cannot better describe than by quoting from Whitehead's recent "Introduction to Mathematics":

"The momentous laws of electric induction were discovered by Michael Faraday in 1831-32. Faraday was asked: 'What is the use of this discovery?' He answered: 'What is the use of a child—it grows to be a man.' Faraday's child has grown to be a man, and is now the basis of all modern applications of electricity. . . . His ideas were extended and put into a directly mathematical form by Clerk Maxwell in 1873. As a result of his mathematical investigations, Maxwell recognized that under certain conditions electric vibrations ought to be propagated. He at once suggested that the vibrations which form light are electrical. This suggestion has since been verified, so that now the whole theory of light is nothing but a branch of the great science of electricity. Also Herz, a German, in 1888, following on Maxwell's ideas, succeeded in producing electric vibrations by direct electrical methods. His experiments are the basis of our wireless telegraphy."

We shall appreciate the important place which mathematics occupies in practical affairs if we try to imagine what would happen if all the contributions which mathematics has made, and which nothing else could make to the progress of engineering, were suddenly withdrawn. The result would obviously be terrific; it would mean nothing less than the total collapse of all industry and commerce, and indeed the complete annihilation of all the external evidences of our material civilization.

"But why," asks the practical man, "do mathematicians and physicists concern themselves so much about certain fields of research which can never, in all likelihood, lead to practical results?" Two good reasons can be given. First of all, truth is one and indivisible; every part of the structure of truth has some bearing on every other part. Sometimes the most theoretical investigation is nearest to the most practical application. Nothing could at first have seemed further removed from the concerns of our daily life than the study of the radiant energy connected with Crooke's tubes, on the one hand, or the use of the so-called imaginary numbers, on the other, and yet look at the practical value of X-rays and of alternating currents, the latter depending essentially on these same imaginary numbers.

Moreover, certain branches of mathematics are no less important because their influence is indirect. In order to gain a thorough understanding of alternating currents we must study the properties of Fourier's series; and, to understand Fourier's series, we must study the theory of functions and of differential equations. These latter, again, depend on various other disciplines, like the theory of equations and the theory of groups. We can never know too much about the space in which we live; hence the practical value of the modern developments of geometry, projective and metrical, analytic and synthetic, algebraic and differential, Euclidean and non-Euclidean, and even *n*-dimensional,—because from one important point of view our ordinary space is four-dimensional.

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* Sibley Journal of Engineering.