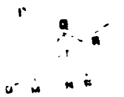
Angle subtended at centre of inscribed circle by an arc between touching points of consecutive sides of polygon $= \frac{360^\circ.3}{10} = 108^\circ$, side $= 2 \tan 54^\circ.1$ ft., and $\tan 54^\circ$ is known, hence solution.



8. Angle $QOS = \frac{1}{2}(\alpha+\beta)$. $POQ = \frac{1}{2}(\alpha-\beta)$. $\therefore$ $POM = \alpha$ and $ROS = \beta$ where angle $QOR = POQ$. PQ is perpendicular to OQ, and RQ is perpendicular to OQ, and PM, QN, RS perpendiculars on OS. $OP = OR$, $MN = NS$.

$$2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta) = 2\frac{ON}{OQ}\cdot\frac{OQ}{OP}$$

$$= \frac{2ON}{OP} = \frac{OM}{OP} + \frac{OS}{OP} = \frac{OM}{OP} + \frac{OS}{OR}$$

$$= \cos\alpha + \cos\beta.$$

$$2\cos(\alpha-\beta)\cos(\theta+\alpha)\cos(\theta+\beta)$$

$$= \cos^2(\theta+\alpha) + \cos(\theta+\alpha)\cos(\theta-\alpha+2\beta)$$

$$= \tfrac{1}{2}\left\{\cos 2(\theta+\alpha) + \cos 2(\theta+\beta) + \cos 2(\alpha-\beta) + 1\right\},$$

with similar values for second and third line; $\therefore$ result is $-\frac{1}{2} + \frac{1}{2}\left\{\cos 2(\beta-\gamma) + \cos 2(\gamma-\alpha) + \cos 2(\alpha-\beta)\right\}$, and is independent of θ

$$= \left\{2\cos(\beta-\gamma)\cos(\gamma-\alpha)\cos(\alpha-\beta)\right\}.$$

10. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$,

$\therefore$ $\log a - \log\sin A = \log b - \log\sin B = \log c - \log\sin C$; differentiate then,

$$\frac{x}{a} - \cot A\,dA = \frac{y}{b} - \cot B\,dB = -\cot C\,dC.$$

$$(1)\quad -dA = -\left(\frac{x}{a} + \cot C\,dC\right)\tan A,$$

$$(2)\quad -dB = -\left(\frac{y}{b} + \cot C\,dC\right)\tan A,$$

and $A + B = \pi - C$, $\therefore$ $-(dA + dB) = dC$,

$\therefore$ from (1) + (2) $-\frac{x}{a}\tan A - \frac{y}{b}\tan B$

$$= dC\left\{1 + \cot C(\tan A + \tan B)\right\}$$

$$[\because \pi - C = A + B] = dC\left\{\tan A \tan B\right\}$$

$$dC = -\left\{\frac{x}{a}\cot B + \frac{y}{b}\cot A\right\}.$$

Let $R = OA$, OB or OC = radius of circle about triangle ABC.

Let R_1 and $R_2 = OQ$ and OP, the radii of circles about triangle BOC and AOC respectively.

And P, Q, R, the centres of the three circles; then

$$R_2 = \frac{AC}{\sin AOC} = \frac{b}{\sin 2B} = \frac{b}{2\sin B\cos B}$$

$$= R\sec B;\ \text{similarly } R_1 = R\sec A.$$

Angle $POQ = A + B$; $PQ^2 = R^2\sec^2 A$

$$+ R^2\sec^2 B - 2R^2\sec A\sec B\cos(A+B)$$

$$= R^2(\tan A + \tan B)^2$$

$$= \frac{R^2}{\cos^2 A\cos^2 B\cos^2 C}\cdot\sin^2 C\cos^2 C.$$

$$\therefore\ \frac{PQ}{\sin 2C} = \frac{R}{2\cos A\cos B\cos C} = \text{etc.}$$

The following problems were sent by W. J. Robertson, M.A., Mathematical Master St. Catharines Collegiate Institute, selected from Wolstenholme's Examples :—

1. The sum of the squares of all the numbers less than a given number N and prime to it is

$$\frac{N^2}{3}\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

$$+\frac{N}{6}(1-a)(1-b)(1-c)\ldots$$

2. The sum of the cubes is

$$\frac{N^3}{4}\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

$$+\frac{N^2}{4}(1-a)(1-b)(1-c)\ldots$$

3. The sum of the fourth powers is

$$\frac{N^4}{5}\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

$$+\frac{N^2}{3}(1-a)(1-b)(1-c)\ldots$$

$$-\frac{N}{30}(1-a^3)(1-b^3)(1-c^3)\ldots$$

where a, b, c are the different prime factors of N.