ARTS DEPARTMENT.

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SOLUTIONS

Of Cambridge Papers—Tripos, 1880, (*see* September number)—by ANGUS MACMURCHY, U.C.

2. (1) Resolve into its component factors

$$(a^3+b^3+c^3)xyz + (b^2c+c^2a+a^2b)(y^2z+z^2x+x^2y) + (bc^2+ca^2+ab^2)(yz^2+zx^2+xy^2) + (x^3+y^3+z^3)abc+3abcxyz.$$

Above expression

$$=(ax+by+cz)(bx+cy+az)(cx+ay+bz).$$

(2) Shew also that if $x+y+z+w=0$, then

$$wx(w+x)^2+yz(w-x)^2+wy(w+y)^2 + zx(w-y)^2+wz(w+z)^2+xy(w-z)^2 + 4xyzw=0.$$

Proposed identity holds, if

$$wx(y+z)^2+yz(w-x)^2+wy(x+z)^2 + zx(w-y)^2+wz(x+y)^2+xy(w-z)^2 + 4xyzw=0$$

if

$$w(yz^2+y^2z+\ldots)+w^2(yz+zx+xy) + xyz(x+y+z)+4xyzw=0.$$

Substituting $-w=x+y+z$ and dividing through by w, this becomes

$$(yz^2+y^2z+\ldots)-(x+y+z)(yz+zx+xy) + 3xyz=0,$$

or $0=0$.

3. Solve the equations:

(ii.) $x^3+y^3=b^3$ (1)

$xy+a(x+y)=ab$ (2)

From (1) $(x+y)\left\{(x+y)^2-3xy\right\}=b^3$, (3)

substituting value of xy from (2) in (3), and putting $x+y=X$

$$X^3-b^3+3aX^2-3abX=0,$$

or $(X-b)(X^2+(3a+b)X+b^2)=0$.

$$\therefore X=b \text{ or } \frac{\pm\sqrt{9a^2+6ab-3b^2}-3a-b}{2},$$

From (2) $x+y=b$

$xy=0$

$\therefore x-y=\pm b$

$\therefore x=b$ or 0, $y=0$ or b,

and other roots may be obtained similarly.

(iii.) $x+y+z=x^2+y^2+z^2$

$=\frac{1}{2}(x^3+y^3+z^3)=3.$

$(x+y+z)^3=27=x^3+\ldots$

$+3(y+z)(z+x)(x+y).$

$\therefore 27=6+3(3-x)(3-y)(3-z).$

$7=27-9(x+y+z)+3(yz+\ldots)-xyz.$

$\therefore xyz=2.$

$yz+zx+xy=3.$

$\therefore z(x+y)+\frac{2}{z}=3.$

$z(3-z)+\frac{2}{z}=3.$

$z^3-3z^2+3z-2=0.$

$z_1^3-1=0$, if $z_1=z-1$.

Roots of last equation are 1 and $\frac{-1\pm\sqrt{-3}}{2}$; &c.

5. If $x_3=\log_{x_1} x_2$, $x_4=\log_{x_2} x_3, \ldots,$

$x_n=\log_{x_{n-2}} x_{n-1}$, $x_1=\log_{x_{n-1}} x_n$,

$x_2=\log_{x_n} x_1$,

then $x_1x_2\ldots x_n=1$.