$2ab+(c-a+b)(c+a-b)$=a positive quantity, since any two of a, b, c are together greater than the third, also a^2m^2, b^2n^2, mn are positive; therefore the whole expression is positive,

21. $(1+x)^n=1+nx+\frac{n(n-1)}{1.2}x^2+$&c.

$+\frac{n(n-1)}{1.2}x^{n-2}+nx^{n-1}+x^n$

$(1-x)^{-3}=1+3x+\frac{3.4}{1.2}x^2+$&c.

The coefficient of x^n in the products of these expressions is

$1+3n+\frac{3.4}{1.2}\frac{n(n-1)}{1.2}+\frac{4.5}{1.2}\frac{n(n-1)n-2}{1.2.3}+$

&c.

Also, $(1+x)^n(1-x)^{-3}=(1-x)^{-3}(2-1-x)^n$

$=(1-x)^{-3}\{2^n-n2^{n-1}(1-x)+$ &c. $\}$

$=2^n(1-x)^{-3}-n2^{n-1}(1-x)^{-2}$

$+\frac{n(n-1)}{1.2}.2^{n-2}(1-x)^{-1}$

+ terms involving powers of $(1-x)$ from o up to $n-3$, none of which can therefore contain x^n. Expanding the first three terms we have

$2^n(1+3x+\frac{3.4}{1.2}x^2+$&c.$+\frac{(n+1)\,n+2)}{1.2}x^{n+})$

$-n2^{n-1}(1+2x+3x^2+$&c.$+(n+1)x^n+)$

$+\frac{n(n-1)}{1.2}2^{n-1}(1+x+x^2+$&c.$+x^n+)$

whence the coefficient of x_n is $=$

$2^n\frac{(n+1)\,n+2)}{1.2}-n2^{n-1}(n+1)+\frac{n(n-1).2^{n-2}}{1.2}$

$=2^{n-3}(n^2+7n+8)$ ∴ &c.

22. The first saves one-third, that is one-fourth and one-twelfth of his salary, and the second saves one-fourth, so that the two together save one-fourth of the whole $4,400, and one-twelfth of the first person's salary; hence this one-twelfth is the difference between $1,310 and $1,100; therefore the first person's salary is $2,520.

23. Two less in the shilling's worth or one less in sixpence worth raises the price a penny a dozen, that is, one less in sixpence worth raises the price of each egg one-twelfth of a penny. Taking this as the unit of price, the price of sixpence worth will be denoted by 72. This number is formed by two factors, one indicating the number of eggs in sixpence worth, and the other the number of units of price in the price of each egg, and these factors are such that if the first be diminished by unity and the second increased by unity, their product is still 72. The factors are therefore 9 and 8; that is, there are 9 eggs in sixpence worth, therefore they are 8 pence a doz. The result may be obtained also from the second factor, for the 8 indicates that the price of each egg is 8 twelfths of a penny, so that the price of a doz. is 8 pence.

24. The greatest possible value of 59.9643+ is 59.96439 or 59.9644; similarly the greatest value of 3962.8+ is 3962.9, therefore the required distance cannot exceed the product of these numbers or 237632.92076. The least values of these numbers are 59.9643 and 3962.8; their product is 237626.52804. The difference between these products is 6.39272, adding half this to the lesser product we obtain 237629.7244, a result which therefore differs from the true distance by less than 3.2 miles.

The following solution is somewhat shorter: Since 59.9643 is true to four places of decimals, therefore all the decimals omitted (being in the 5th, 6th, &c., places), are together less than 1 in the 4th place, that is, are less than .0001; therefore the error caused by taking only four places of decimals in 59.9643 is less than .0001, and therefore the error in the product of the two numbers is less than 3962.8 × .0001, that is less than .39628. Again the error caused by taking 3062.8 (omitting the digits in the 2nd, 3rd, &c., decimal places), is less than .1, and therefore the error in the product, due to this omission, is less than 59.9643 × .1, that is, less than 5.99643; adding to these two errors the product of .0001 and .1 we have the total error in the product less than 6.39272. Adding half of this number to the product of the given numbers gives the same approximate result as before.

25. P Q R

Let R be the point when A overtakes B; then since A goes the distance PR while B goes QR, A's rate is to B's as PR : QR, and therefore in going equal distances B's time : A's time as PR : QR. Now A takes 4 hrs. in