$$(x+z)^n = (x+y)^n + \frac{(x+z)^n - (x+y)^n}{(x+z)-(x+y)}\,(z-y).$$

For *convenience* write $f(x)$ for $\frac{(x+z)^n - x^n}{(x+z)-x}$, then $f(x+y)$ represents $\frac{(x+z)^n-(x+y)^n}{(x+z)-(x+y)}$, thus

$$(x+z)^n = x^n + f(x)z \qquad (8)$$

$(x+z)^n = (x+y)^n + f(x+y)(z-y)$. Expand by (5) and (7) and arrange in terms of $y$.

$$(x+z)^n = x^n + f(x)z + \left\{ nx^{n-1} + f'(x)z - f(x) \right\} y + R\,(y^2, y^3, \text{- - -})$$

For this to remain an *identity* the coefficients of $y$, $y^2$, &c., must vanish identically.

$$\therefore\ nx^{n-1} + f(x)z - f(x) = 0. \qquad (9)$$

Substitute $(x+y)$ for $x$, and $(z-y)$ for $z$, expand by (5) and (7), and arrange in terms of $y$.

$$\therefore\ nx^{n-1} + f'(x)z - f(x) + \left\{ n(n-1)x^{n-2} + f''(x)z - 2f'(x) \right\} y + R\,(y^2, y^3, \text{- - -}) = 0$$

For the left hand member to *vanish identically*, the coefficients of $y$, $y^2$, &c., must vanish identically,

$$\therefore\ n(n-1)\,x^{n-2} + f''(x)z - 2f'(x) = 0 \qquad (10)$$

Repeat the operations and the reasoning by which (10) was obtained from (9) and $n\,(n-1)\,(n-2)\,x^{n-3} + f'''(x)z - 3f''(x) = 0$ (11)

$$n\,(n-1)\,(n-2)\,(n-3)\,x^{n-4} + f^{iv}(x)\,z - 4f'''(x) = 0 \qquad (12)$$

and by 'induction,'

$$n\,(n-1)\,(n-2)\,(n-3)\ \text{- - - -}\ (n-m+1)\,x^{n-m} + f^m(x)\,z - mf^{m-1}(x) = 0. \qquad (13)$$

Commencing with (9) and proceeding through (10), (11), (12,) - - - (13), substituting successively for $f(x)\,f'(x)\,f''(x)$, - - - - -, in (8) and the resulting identities,

$$(x+z)^n = x^n + nx^{n-1}z + f'(x)z^2$$

$$= x^n + nx^{n-1}z + \frac{n\,(n-1)}{1.2}x^{n-2}z^2 + \frac{f''(x)\,z^3}{1.2}$$

$$= x^n + nx^{n-1}z + \frac{n\,(n-1)}{1.2.}x^{n-2}z^2 + \frac{n\,(n-1)\,(m-2)}{1.2.3}x^{3n-3}z + \frac{f'''(x)\,z^4}{1.2.3.}$$

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$$= x^n + nx^{n-1}z + \frac{n\,(n-1)}{1.2}x^{n-2}z^2 + \frac{n\,(n-1)\,(n-2)}{1.2.3}x^{n-3}z^3 + \text{- - -} + \frac{n\,(n-1)\,(n-2)\ \text{- - -}\ (n-m+1)}{\underline{|m}}x^{n-m}z^m + \frac{f^m(x)\,z^{m+1}}{\underline{|m}} \qquad (14)$$

This, with the 'remainder' omitted, is the Binomial Theorem in the form in which it is usually given.

If $\frac{f^m(x)}{\underline{|m}}$ be developed each term after the first will be numerically between the corresponding terms of

$\frac{n\,(n-1)\,(n-2)\ \text{- - -}\ (n-m)}{\underline{|m+1}}(x+oz)^{n-(m+1)}$ and

$\frac{n\,(n-1)\,(n-2)\,(n-m)}{\underline{|m+1}}(x+z)^{n-(m+1)}$ and the signs

(affections) will be the same, so we may assume $\frac{f^m(x)}{\underline{|m}}$

$= \frac{n\,(n-1)\,(n-2)\ \text{- - -}\ (n-m)}{\underline{|m+1}}(x+pz)^{n-(m+1)}$ *in which* p *is some proper fraction*. This is *a* form into which the remainder may be thrown.

If this form be substituted in the identity (14) it will give the expansion

$$(x+z)^n = x^n + \frac{n}{1}x^{n-1}z + \frac{n\,(n-1)}{1.2}x^{n-2}z^2 + \text{- - -} + \frac{n\,(n-1)\,(n-2)\ \text{- - -}\ (n-m+1)}{\underline{|m}}x^{n-m}z^m + \frac{n\,(n-1)\ \text{- - -}\ (n-m)}{\underline{|m+1}}(x+pz)^{n-m-1}z^{m+1}$$

This last is not an identity, but if the right arithmetical value be given to $p$, the series will be *arithmetically equal* to the binomial.

In the above, $f(x)$, &c., have been used *merely for convenience;* if their actual values be substituted for them, (9), (10), - - - (14) will be *seen* to be identities. In this method (6) and (7) will not be needed.

## I. Papers on Practical Education.

### 1. THE TEACHER'S VOICE.

Did you ever watch children at their favorite game of "Playing School?" If so, you must have observed that the child who personates the teacher is sure to issue his numerous orders in a peculiarly harsh and shrill tone of voice. The reason why is not far to seek. The little one is shrewdly observant of his elders, and has come to associate with the pedagogic business a harsh and artificial utterance.

A sweet and well-modulated voice is one of the teacher's best possessions; calm, full, and low pitched, it is a great aid in school discipline. Careful culture will do much to improve the quality and compass of the voice. We commend to the careful perusal of our readers the following entertaining and valuable essay by a distinguished English writer:

Far before the eyes, or the mouth or the habitual gesture, as a revelation of character, is the quality of the voice, and the manner of using it. It is the first thing that strikes us in a new acquaintance, and it is one of the most unerring tests of breeding and education. There are voices which have a certain truthful ring about them—a certain something, unforced and spontaneous, that no training can give. Training can do much in the way of making a voice, but it can never compass more than a bad imitation of this quality; for the very fact of its being an imitation, however accurate, betrays itself, like rouge on a woman's cheeks, or a wig, or dyed hair. On the other hand, there are voices which have the jar of falsehood in every tone, and that are as full of warning as the croak of the raven, or the hiss of the serpent. There are, in general, the naturally hard voices, which make themselves caressing, thinking by that to appear sympathetic; but the fundamental quality strikes through the overlay, and a person must be very dull indeed who cannot detect the pretence in that slow, drawling, would-be-affectionate voice, with its harsh undertone and sharp accent, whenever it forgets itself. But, without being false or hypocritical there are voices that puzzle as well as disappoint us, because so entirely inharmonious with the appearance of the speaker, For instance, there is that thin treble squeak we sometimes hear from the mouth of a well-grown, portly man, when we expected the fine rolling utterance which would have been in unison with his outward seeming; and, on the other side of the scale, where we looked for a shrill head voice, or a tender musical cadence, we get that hoarse chest voice, with which young and pretty girls will sometimes startle us.

Nothing betrays so much as the voice, save, perhaps, the eyes, and they can be lowered, and so far their expression hidden. In moment of emotion, no skill can hide the fact of disturbed feelings, though a strong will and the habit of self-control can steady the voice when else it would be failing and tremulous. But not the strongest will, nor the largest amount of self-control, can keep it natural as well as steady. It is deadened, veiled, compressed, like a wild creature, tightly bound and unnaturally still. One feels that it is done by an effort, and that if the strain were relaxed for a moment, the wild creature would burst loose in rage or despair, and the voice would break out into the scream of passion, or quiver away into the falter of pathos. And this very effort is as eloquent as if there had been no holding down at all, and the voice had left to its own impulse, unchecked. Again, in fun and humor, is it not the voice that is expressive, even more than the face? The twinkle of the eye, the hollow in the under lip, the dimples about the mouth, the play of the eyebrow, all are aids, certainly; but the voice! The mellow tone that comes into the utterance of one man, the surprised accents of another, the fatuous simplicity of a third, the philosophical acquiescence of a fourth, when relating the most outrageous impossibilities—a voice and manner peculiarly transatlantic, and, indeed, one of the Yankee forms of fun—do not we know all these varieties by heart? Have we not veteran actors, whose main point lies in one or other of these varieties? And what would be the drollest anecdote, if told in a voice which had neither play nor significance? Pathos, too,—who