point and take up a position in the interior of the arch, as such a position would correspond with a greater amount of thrust at the key. The point D being determined we know in what direction the arch will move if the half arch is for a moment left unsupported. The path which each point in the arch tends to take is evidently the arc of a circle around D as a centre. Thus the points A and B will tend to describe the small arcs AA' and BB' corresponding to the same angle at the centre D. The resistance of the other half of the arch however prevents actual motion; but it follows that the points A and B support unequal pressures, as the amount of compression at these points is evidently proportional to the horizontal distances through which they would move respectively, if they were free to do so. The effect produced by the other half of the arch is to maintain the joint AB in a vertical position; and it follows that the horizontal compression at each point is proportional to the decrease in length of the horizontal projections of the radii drawn from D to A and B respectively. It is easily shown that this proportion is the same as the ratio of $h+t$ to h. If we take (Fig. 5.) two lines AP and BQ in this ratio, it follows from the accepted theory of elasticity that the resultant of the whole pressure on AB will pass through the centre of gravity of the trapezium $APQB$. The point K is therefore not at the centre of AB, but always a little above it. For the limiting cases, if $h=o$ as in the *plate bande* or straight arch, $BK=\frac{2}{3}t$; and if $h=$infinity, $BK=\frac{1}{2}t$. This reasoning assumes that the point D is rigidly fixed, or in other words that the abutments are immovable and incompressible; and here we meet with the uncertainty mentioned at the outset. If the point D were to yield laterally, the point K would rise in proportion; and would approach as near to A as the strength of the material allowed.

The figure employed in the demonstration is for a segmental arch; but for a full arch it can be shown by similar considerations that the curve of pressure must be tangent to the intrados where it meets it, and that the point of contact determines the position of the joint of rupture. In the actual arch the curve must be sufficiently within the intrados at the joint of rupture to prevent crushing. He further shows that the positions which the joint of rupture can take are always between one half and two-thirds of the rise, in the case both of the semi-circle and ellipse. The exact position depends on the ratio of the thickness of the arch ring to the rise of the arch, as this affects the distance of the centre of gravity from the key: but for the ordinary proportions found in practice, the joint will be very nearly at half the rise.

Dupuit's conclusion is then that with unyielding abutments the position of the curve of pressure will be determined as follows:—

The point K will be at the centre of A B for a semi-circular or elliptical arch, and at the upper third of A B for a plate-bande. For intermediate forms it will lie proportionately between these limits.

For semi-circular and elliptical arches the curve of pressure will be tangent to the intrados at the joint of rupture, and the joint of rupture is to be determined by this condition. For segmental arches of less amplitude than the arc between the joints of rupture in the full arch, the curve of pressure will pass through the point D at the springing.

In examining this theory it is evident that the determination of the point K would not be appreciably affected if the point J were to move out along the joint of rupture; but on the other hand it does depend entirely on the supposition that the abutments are unyielding. If the abutments give laterally under the effect of the thrust, the joints of rupture will open and also the joint at the key. The curve of pressure will then necessarily pass through A and D; but the value of the thrust will be considerably less than for a curve passing through K and D. We see then that any yielding of the abutments gives an immediate reduction in the value of the trust; and therefore the abutments require greater strength to resist yielding than to stand after a slight motion has taken place. This accounts for the regained stability of structures in which a slight movement has occurred. We have also in this a distinguishing difference between the theories of Dupuit and Scheffler. As examples of cases in which the condition of unyielding abutments becomes perfectly realized, we may mention a segmental arch springing from the solid rock, or a series of arches all equal and equally loaded; but it will be practically fulfilled in any case in which the abutments themselves and their foundations are sufficiently resisting.

It cannot be too carefully noted, however, that Dupuit's reasoning is based upon the consideration of the arch ring itself while standing alone. For that case then we must consider his conclusions as established. But in supposing that the curve of pressure will remain tangent to the intrados when the arch ring carries the weight of the spandrel walls, backing and filling, Dupuit is certainly going beyond the limits of what he has actually proved; although he renders most valuable assistance to the consideration of the subject by indicating the true starting point and the first steps to be taken before more complicated conditions can be examined with advantage. His method for the completed arch is to find by trial a new position for the joint of rupture which will fulfill the condition of intersection, the tangent at the joint and the thrust at the key having to intersect on the gravity