This will continue for n terms, and we have the sum equal to

$$1+2n+\frac{2n(2n+1)}{|\underline{2}}+\ldots+\frac{2n}{(|\underline{n})^2}, \text{ but}$$

$$(1+1)^{2n}=1+2n+\frac{2n(2n-1)}{|\underline{2}}+\ldots$$
$$+\frac{2n}{(|\underline{n})^2}+\ldots+2n+1;$$

if we add $\dfrac{2n}{(|\underline{n})^2}$ to each side of this latter we have double the number of combinations;

$\therefore$ twice the number

$$=(1+1)^{2n}+\frac{|\underline{2n}}{(|\underline{n})^2}=2^{2n}+\frac{|\underline{2n}}{(|\underline{n})^2},$$

or the number $=2^{2n-1}+\dfrac{|\underline{2n}}{2(|\underline{n})^2}$.

PROBLEMS.

176. Apply the principles of algebraic expansion and factoring to the solution of the following arithmetical problems:—

Simplify

(a) $\dfrac{\frac{1}{4}-\frac{1}{3}+\frac{1}{9}}{\frac{1}{2}-\frac{1}{3}}$; (b) $\dfrac{(\frac{3}{2}+\frac{1}{2})^2-\frac{3}{8}-\frac{1}{3}+\frac{1}{4}}{\frac{3}{8}+\frac{1}{3}-\frac{1}{2}}$;

(c) $\dfrac{2499}{49}$; (d) $\dfrac{6364}{15\times5+11}$;

(e) $\dfrac{2\frac{3}{7}+1+\frac{9}{8}+\frac{27}{84}}{(\frac{2}{3}+\frac{3}{4})^2}$; (f) $\dfrac{(\frac{1}{2})^4+2(\frac{1}{2}\times\frac{1}{3})^2+(\frac{1}{3})^4}{\frac{1}{4}+\frac{1}{9}}$;

(g) $\dfrac{.5\times.25-25\times.0625}{.5-(5)^2}$; (h) $\dfrac{2\times4\times8-.5\times.25}{5-1.5}$

177. Factor

(a) $(1+x)^2+(1+x^2)^2+2(1+x^4)+2x(1+x^2.)$

(b) $(x+y+z)^2+(x-y-z)^2+2x^2-2(y+z)^2$.

(c) $(1+2x+x^2)^2+(1-2x+x^2)^2+2(1-x^2)^2$.

(d) $(x\times y)^4-5(x^2+y^2)-10xy-24$.

(e) $p+q+r(p+q+1)+s(1-p-q)+r^2-s^2$.

(f) $x^2+y^2+x+y+2xy-xz-yz$.

(g) $p+q+r(p+q+r+s)-s(p+q+r+s)+r+s$.

178. Shew that

$$(x+a)^3+(x-a)^3=2(x^3+3a^2x);$$

also that

$$(x+a)^3-(x-a)^3=2(3ax^2+a^3),$$

and from these formulæ simplify

(a) $(a+b+c)^3+(a+b-c)^3$;

(b) $(a+b+c)^3-(a-b-c)^3$;

(c) $(x+y+1)^3-(x+y-1)^3-2\{3(x+y)^2+1\}$.

179. Simplify

(a) $(x+a+b)^3+(x+a-b)^3$
$\qquad+6(x+a)(x+a)^2-6b^2(x+a)$;

(b) $8(x+a+b)^3-(2x+2a)^3$
$\qquad-8b^3-24b(x+a)(x+a+b)$.

180. If a cubic foot of water weighs 1,000 ounces, and the specific gravity of silver be 10.5, find how many ounces of silver would be required to make an inkstand, in the form of the frustrum of a regular hexagon, 4 inches high, each of whose sides at the base is 2 inches long and at the top 1 inch long, the *hollow* being in the form of a right cylinder, extending to within *one* inch of the bottom, and arranged about the central axis, so as to leave a wall ¼ of an inch thick at the middle of each side at the top.

181. *A* owes \$4,000, due in three years, bearing interest at 6 per cent. per annum. He wishes to make equal half-yearly deposits in the bank, so that at the end of the three years, these deposits, with accrued interest, may be just sufficient to cancel the debt, the bank allowing interest at 5 per cent. per annum, payable half yearly, and the first deposit to be made at the end of the first half year. Just after making his payment at the end of the second year he is compelled to draw out of the bank \$1,000; find how much each of his last two payments must be increased on this account; also find the total amount of the last deposit.

182. A brass scale of a barometer has been correctly graduated at 62° Fahr.; find the true reading of the barometer when it shews 30 inches at 87° Fahr., corrections being made for the expansions of the scale and the mercury, the co-efficient of expansion of brass being .00001 for every degree Fahr., and one vol. of mercury at freezing point (+32°) occupying 1.0054 vols. at 87°.

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