(*b*). Find the co-efficient of x^n in the expansion of $\frac{1+x+x^2}{1-2x+x^2}$.

This is $(1+x+x^2)(1-x)^{-2} = (1+x+x^2)(1+2x+3x^2+\ldots(n-1)x^{n-2}+nx^{n-1}+\overline{n+1}.x^n+\ldots)$

And the co-efficient of x^n is $n-1+n+n+1$, or $3n$.

This may also be done as follows, and, although not so convenient in this particular case, the expansion might be more convenient in many cases.

$$\frac{1+x+x^2}{1-2x+x^2} = \frac{\frac{1-x^3}{1-x}}{(1-x)^2} = \frac{1-x^3}{(1-x)^3}$$

$$= (1-x^3)\left[1+{}^3h_1x+{}^3h_2x^2+\ldots{}^3h_{n-3}x^{n-3}+\ldots{}^3h_nx^n+\ldots\right]$$

And the co-efficient of x^n is ${}^3h_n - {}^3h_{n-3}$.

Where ${}^ph_r =$ the number of homogeneous products of r dimensions which can be made from p letters

Then ${}^3h_n = \frac{(n+2)!}{2\,.\,n!} = \frac{1}{2}(n+1)(n+2)$

${}^3h_{n-3} = \frac{(n-1)!}{2(n-3)!} = \frac{1}{2}(n-1)(n-2)$

$\therefore$ Co-efficient $= \frac{1}{2}[(n+1)(n+2)-(n-1)(n-2) = 3n$

(*c*). If $(1+x)^n = 1+c_1x+c_2x^2+\ldots c_nx^n$, show that

$1-c_1^2+c_2^2-+\ldots-c_n^2 = 0$, n being an odd integer.

If $(1+x)^n = 1+c_1x+c_2x^2+\ldots c_nx^n$, then $(1-x)^n = 1-c_1x+c_2x^2-+\ldots-c_nx^n$

The co-efficient of x^n from the product of the right-hand members is

$1^2-c_1^2+c_2^2-c_3^2+-\ldots-c_n^2$.

And from the product of the left-hand members it is the co-efficient of x^n in $(1-x^2)^n$. But if n is odd, this co-efficient is zero, since the expression contains only even powers of x. $\therefore\ 1^2-c_1^2+c_2^2-c_3^2+-\ldots-c_n^2=0$.

8. A mortgage of A dollars bearing interest at a per unit per annum, payable yearly, has b years to run. Find its present value, money being worth r per unit per annum, payable half yearly.

If the payments of interest on the mortgage be deposited in a bank until the b years are up, and the present value be deposited for the same length of time, the accumulated sums should be the same in each case. And in justice we must treat the interest payments as present money deposited at compound interest at r per unit payable half yearly.

The last payment of interest amounts to Aa.

The payment before the last " " A$a(1+{}^r/_2)^2$.

The first payment amounts to A$a(1+{}^r/_2)^{2b-2}$.

$\therefore$ The whole deposit of interest at the end of b years is

$$Aa\left[1+(1+{}^r/_2)^2+(1+{}^r/_2)^4+\ldots(1+{}^r/_2)^{2b-2}\right] = aA\,.\,\frac{(1+{}^r/_2)^{2b}-1}{(1+{}^r/_2)^2-1}.$$

And adding the face of the mortgage, which is payable with the last payment of interest, we have, for the sum accruing from the mortgage,

$$A\left\{1+a\,.\,\frac{(1+{}^r/_2)^{2b}-1}{(1+{}^r/_2)^2-1}\right\}.$$

And if V be the present value, its accumulated value at the end of b years is V . $(1+{}^r/_2)^{2b}$.

Equating these gives $V = A\left\{1+a\,.\,\frac{(2+r)^{2b}-2^{2b}}{(2+r)^2-2^2}\,.\,\frac{2^2}{2^{2b}}\right\}\left(\frac{2}{2+r}\right)^{2b}$