from the circular curve outward to every corresponding point in the equal distance $C M$, are equal in magnitude and distribution, and $\therefore D M$ is equal to $M P$ and half of $D P$ (Fig. 13). Hence the offset or shift $D P$ (= O), and the transition curve $A M C$ bisect each other at M.

(3) The offsets from a tangent to a circular curve vary as the square of the distance from the tangent point (nearly), or to formulate it O varies as $n^2 D$.

Where O = offset from tangent,
n = distance of the offset from the tangent point,
D = degree of curve,

but by our definition of a transition curve the degree of curve at any instant also varies as n. Therefore in a transition curve of this nature O varies as $n^2 \times n = n^3$......(12), and also by paragraph (2). If we have a given offset from tangent to circular curve at $D = D P$ (= O), then the offset to the transition curve at a distance m from A is equal to $\left(\frac{m}{n}\right)^3 \frac{O}{2}$ where $n = \frac{1}{2}$ length of transition = $A M$ = $M C$. And, in the same way, measuring back from C along the curve toward P at any distance, m the offset outward from the circular curve to the transition curve =

$$\left(\frac{m}{n}\right)^3 \frac{O}{2} \text{ .... (13)}$$

The equation to the cubic parabola can now be established in terms of the offset O and $\frac{1}{2}$ length n. let $y = C x^3$, but when $y = \frac{O}{2}, x = n$, therefore $C = \frac{O}{2} \times \frac{1}{n^3}$

$$\text{and } \therefore y = \frac{O}{2n^3} x^3 \text{ .... (14)}$$

(4) because by equation (12) offsets to a transition curve vary as cube of distance from origin, therefore in Fig. (13) $C C^1 = 8 \times D M = 4 O$, and therefore $G C = 3 O$.........(15)

Now, for very small angles, $G C = P C \times \sin \frac{X}{2}$ (nearly), and $P C = 2 \times I C \times \sin \frac{X}{2}$ (nearly), therefore by substitution we get

$G C = 3 O = 2 I C \times \sin^2 \frac{X}{2}$ (nearly), but $I C = \frac{5730}{D}$ (D = degree of curve), and $\therefore O = \frac{3820}{D} \times \sin^2 \frac{X}{2}$ and

$$\sin \frac{X}{2} = \sqrt{\frac{O \times D}{3820}} = .01618 \sqrt{O \times D} \text{ ....(16)}$$

from which we can get X, having O and D; or otherwise, since $n = \frac{X}{D}$ (evidently), and for small angles $\sin \frac{X}{2}$ = .0087 X (in degrees).

$\therefore$ substituting in (16), we get

$$X = \frac{.01618}{.0087} \sqrt{O \times D} = 1.86 \sqrt{O \times D} \text{ .... (17)}$$

and $n = \frac{1}{2}$ length of transition $= 1.86 \frac{\sqrt{O \times D}}{D} =$

$$1.86 \sqrt{\frac{O}{D}} \text{ .... (18)}$$

This can also be put in the approximate form,

$$O = \frac{n^2}{6R} \text{ .... (19)}$$

Where R = radius of the curve.

Equations (12) to (19) give such relations between X, n, O and D as will enable any length of transition curve to be put in for any degree of curve.

(*e.g.*) Let D = 10° curve, and O = 10 feet.

Substituting in (18) we get, n = 186 feet, or the transition is $2n$ = 372 feet, which is somewhat longer than is needed.

(*e.g.*) Let X = 15°, and D = 10° curve.

Then, $O = \frac{15 \times 15}{(1.86)^2 \times 10} = 6.5$ feet, and $n = 100 \times 1.86 \sqrt{\frac{6.5}{10}} =$ 150 feet, which latter could have been determined directly. Also $\frac{O}{2} = \frac{6.5}{2} = 3.25$ feet, and any other offset will vary as cube of distance from A; that at the quarter points being, for instance, $(\frac{1}{2})^3 \times 3.25$ = .41 feet.

A most usual length of transition is 30 feet per degree of curve, which permits of the super elevation being lowered at $\frac{1}{2}$ inch per 30 feet = 1 rail length, which is a most usual amount.

Now, although these equations enable us to put in transitions by offsets, if we have for instance, the tangents already in place, and can move the main curves inward bodily so as to permit the requisite "shift" O, which is very useful if, on construction, the rigid curves and tangents are found already in place, and offsetting is the quickest method to use—still, we also wish to be able to put in transitions as a regular part of location, and not as an afterthought, and to do so it is necessary to determine methods of locating such curves by transit deflections from the beginning, end, or intermediate points.

Any small angular deflection from a meridian to any point varies as $\frac{\text{offset}}{\text{distance}}$, or in other words the natural tangent of any small angle is its circular measure.

Now referring to equation (12) and Fig. (13) any offset from the tangent $A C$ to the transition curve varies as the cube of the distance from A.

$\therefore$ angular deflections to the transition curve from tangent A C, using A as origin, vary as $\frac{\text{offset}}{\text{distance}}$ =

$$\frac{(\text{distance})^3}{\text{distance}} = (\text{distance})^2 \text{ .... (20)}$$

Also in Fig. (13) $\frac{G C}{A C} = \frac{1}{2} \frac{G C}{P C} = \frac{1}{2} \times \frac{X}{2} = \frac{3 \times O}{2 \times n}$ (evidently)

$\therefore X = \frac{6 \times O}{n}$ but the angle $C'AC = \frac{C' C}{C A} = \frac{4 \times O}{2 \times n} = \frac{2 \times O}{n}$

$$\therefore \text{ the angle } C'A C = \frac{1}{3} \times \frac{6 \times O}{n} = \frac{1}{3} X \text{ ...... (21)}$$

Equations (20) and (21) enable us to determine any deflections to the transition curve from the point A; (*e.g.*) let a 10° curve have a transition curve 300 feet long then $X = \frac{n}{D} = \frac{300}{2} \times \frac{1}{10} = 15°$.

$\therefore$ by (21) the angle $C'A C$ = 5° = 300′ = deflection from tangent at A to the end of the transition, and by equation (20) the deflections to each 30′ intermediate point are:

1st 30 ft. point $\left(\frac{30}{300}\right)^2 \times 300' = 03'$

2nd " $\left(\frac{60}{300}\right)^2 \times 300' = 12'$

3rd " $\left(\frac{90}{300}\right)^2 \times 300' = 27'$

4th " $\left(\frac{120}{300}\right)^2 \times 300' = 48'$

etc., etc., etc., etc.

This series of deflections from the origin A, continued as far as necessary, may be called *a foundation series*, and is the basis of all deflections forward or backward from any point. We must now, in order to fix on intermediate deflections, with the transit also at some intermediate point, look on a transition curve, thus: (See Fig. 15). Suppose it to be stopped at 1, then it is a transition curve to a 1°