(i) $(1+x)^{\frac{1}{2}}+(1-x)^{\frac{1}{2}}=2^{\frac{1}{2}}$.

(ii) $\begin{cases} \frac{x}{9}+\frac{y}{8}=48 \\ \frac{x}{8}+\frac{y}{9}=42 \end{cases}$

(iii) $\begin{cases} 7yz=9(y+z) \\ xz=8(z+x) \\ xy=8(x+y) \end{cases}$

7. A messenger starts on an errand at the rate of 4 miles an hour; another is sent $1\frac{1}{2}$ hours after to overtake him; the latter walks at the rate of $4\frac{1}{4}$ miles an hour; when and where will he overtake him?

The road from a place A to a place B first ascends for 5 miles, is then level for 4 miles, and afterwards descends for 6 miles, the rest of the distance; a man walks from A to B in 3 hours 52 min.; the next day he walks back to A in 4 hours, and he then walks half way to B and back again in 3 hours 55 minutes; find his rates of walking up hill, on level ground, and down hill.

8. Solve

(i) $x^2-2x+6(x^2-2x+5)^{\frac{1}{2}}=11$

(ii) $\begin{cases} x^4+y^4=257 \\ x+y=5 \end{cases}$

(iii) $\begin{cases} a^xb^yc^z=l \\ a^yb^zc^x=m \\ a^zb^xc^y=n \end{cases}$

9. From a given point draw a straight line equal to a given straight line.

10. If one side of a triangle be produced, the exterior angle is greater than either of the interior opposite angles.

From a given point there can be drawn to the same straight line only two straight lines equal to one another.

11. If a straight line touches a circle, and from the point of contact a straight line be drawn cutting the circle, the angles made by this line with the line touching the circle shall be equal to the angles which are in the alternate segments of the circle.

P and Q are two points in the circumference of two concentric circles. The angle included between the tangents at P and Q is equal to that subtended at the centre by PQ.

SOLUTIONS.

1. $\frac{397\frac{5}{7}}{11\frac{2}{7}} \div \frac{262\frac{4}{13}}{8\frac{1}{3}} = \frac{91}{81}$.

3. Amt. of stock $\times \frac{1\frac{1}{2}}{100}$ + amt. of stock $\times \frac{1\frac{1}{2}}{100} \times \frac{1\frac{1}{2}}{90} = 457.5$.

$\therefore$ amount of stock = 30,000.

$\therefore$ excess required $= 3000 \times \frac{1\frac{1}{2}}{100} \times \frac{1\frac{1}{2}}{90} =$ £7 10s.

4. (1). $\frac{cx+d}{ax+b}$. (2) $\frac{x+2}{x+3}$. (3) $\left(\frac{y}{x}\right)^{\frac{1}{3}}$

5. (1). x^3+3x^2+5x+5. (2). $3x^2-4x+2$.

6. (1). Cubing both sides

$$1+y+3(1-x^2)^{\frac{1}{3}}\{(1+x)^{\frac{1}{3}}+(1-x)^{\frac{1}{3}}\}+1-x=2,$$

or $3(1-x^2)^{\frac{1}{3}}(2^{\frac{1}{3}})=0$, or $x=\pm 1$.

(2). 144, 216.

(3). Equations become

$$7=\frac{9}{z}+\frac{9}{y}, \text{ &c. } x=-\frac{114}{23},\ y=\frac{144}{71},\ z=\frac{144}{41}.$$

7. If x be the time $4x=(x-1\frac{1}{2})\,4\frac{2}{3}$, or $x=9\frac{1}{2}$.

If x, y, z be the rates of walking up hill, on the level, and down hill, $\frac{5}{x}+\frac{4}{y}+\frac{6}{z}=3\frac{1}{60}$, &c. $x=3,\ y=4,\ z=5$.

8. (1). Completing square $x^2-2x+5+6\sqrt{x^2-2x+5}+9=25$.

$\therefore x^2-2x+5=4$ or 64,

$\therefore x=1$, or $1\pm 2\sqrt{15}$.

(2). Squaring the second and subtracting $xy(4x^2+8xy+4y^2-2xy)=868$, or $xy(100-2xy)=868$; whence $xy=46$ or 4. Then

$x+\frac{46}{x}=5$; $\therefore x=\frac{1}{2}(5\pm\sqrt{-159})$. Also, $x+\frac{4}{x}=5$;

$\therefore x=4$ or 1.

(3). Taking logarithms $x\log a+y\log b+z\log c=\log l$, &c.

Let O be the common centre, A the point where the tangents intersect, and K the intersection of PA, OQ. Then angles at P and Q are equal; also angles PKO, QKA; $\therefore$ angles POQ, PAQ are equal.

EUCLID—HONORS.

Examiner—F. Hayter, B.A.

1. The angles which one straight line makes with another upon one side of it are either two right angles or are together equal to two right angles.

2. If the square described upon one of the sides of a triangle be equal to the squares described upon the other two sides of it, the angle contained by these two sides is a right angle.

3. If a straight line be divided into any two parts, the rectangles contained by the whole and each of the parts are together equal to the square on the whole line.

4. Describe a square which shall be equal to a given rectilineal figure.

5. Find the centre of a given circle.

6. If two circles touch each other externally in any point, the straight line which joins their centres shall pass through that point of contact.

7. From a given circle cut off a segment which shall contain an angle equal to a given rectilineal angle.

8. Inscribe a circle in a given triangle.

9. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides, or these produced, proportionally.

10. Triangles which have one angle in the one equal to one angle in the other, and their sides about the equal angles reciprocally proportional are equal to one another.

11. Rectilineal figures which are similar to the same rectilineal figure are also similar to one another.

12. If one angle of a triangle is equal to the sum of the other two, the greatest side is double of the distance of its middle point from the opposite angle.

13. One of the diagonals of a parallelogram being given, and the angle which it makes with one of the sides, complete the parallelogram, so that the other diagonal may be parallel to a given line.

14. Describe a rectangle equal to a given square, and having one of its sides equal to a given straight line.

15. Two points are taken in the diameter of a circle at equal distances from the centre. Through one of these draw any chord, and join its extremities and the other point. The triangle so formed has the sum of the squares of its sides invariable.

16. If ABC is a triangle described in a circle, and the tangent at A meets BC produced in D, prove that

$$CD : BD = CA^2 : BA^2$$