V. If the 1st of March be called the first day of the year, shew that if the pth day of the qth month be the nth day of the year,

$$n = 30\overline{q-1} + p + r$$

where r is the greatest integer in $\frac{5q+4}{12}$.

VI. If p, q, and r be three consecutive primes to 3, prove that

$$p\,(p-2q) - r\,(r-2q) = \pm\,3,$$

the upper or lower sign being taken according as q exceeds p by 2 or 1.

VII. If $\frac{1}{1-2rx+x_2} = 1 + b_1 x + b_2 x^2 + \ldots$ $b_n x^n + \ldots$ prove that $b_{n+1} \cdot b_{n-1} = b_n^2 - 1$.

VIII. Shew that the sum of the series,

$$\frac{\lfloor n+1}{\lfloor n-r-p\ \lfloor r+p+1} + r \cdot \frac{\lfloor n+1}{\lfloor n-r-p+1\ \lfloor r+p} + \ldots$$

$$\ldots + \frac{\lfloor r}{\lfloor s\ \lfloor r-s} \cdot \frac{\lfloor n+1}{\lfloor n-r-p+s\ \lfloor r+p-s+1} + \ldots$$

$$= \frac{\lfloor n+r+1}{\lfloor r+p+1\ \lfloor n-p}.$$

IX. Solve the equation $(x-3)\ (x-9)\ (x-11)\ (x-17)$.

$$= (x-8)\ (x-14)\ (x-16)\ (x-22).$$

X. $$x + y + z = 11$$
$$xy + yz + zx = 36$$
$$yz = 3x\,(z-y).$$

One solution is $x=2$, $y=3$, $z=6$; find all the other solutions.

11. Prove that $\sin 60° = 4 \sin 20° \sin 40° \sin 80°$.

XII. Eliminate θ between the equations.

$$m = \operatorname{cosec}\theta - \sin\theta$$
$$n = \sec\theta \cos\theta.$$

XIII. In any triangle, prove that

$$\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$$

$$= \frac{1}{3}\left\{\left(\frac{a}{\sin A}\right)^2 + \left(\frac{b}{\sin B}\right)^2 + \left(\frac{c}{\sin C}\right)^2\right\}$$

$$\left\{\left(\frac{\sin^2\frac{A}{2}}{\frac{a}{2}}\right)^2 + \left(\frac{\sin^2\frac{B}{2}}{\frac{b}{2}}\right)^2 + \left(\frac{\sin^2\frac{C}{2}}{\frac{c}{2}}\right)^2\right\}$$

XIV. The circumference of a circle whose radius is a is divided into n points, each of which subtends the same angle at a point O within the circle. If $CO = b$ and $r_1, r_2, \ldots r_n$, be the lines from O to the points of division, shew that

$$r_1 + r_2 + \ldots\ldots + r_n = (a^2 - b^2)\left(\frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots + \frac{1}{r_n}\right).$$

XV. A circle is inscribed in a triangle, and a second triangle is formed whose sides are equal to the distance of the points of contact from the angles of the triangle; if r be the radius of the circle inscribed in the first triangle, and p, p', the radii of the inscribed and circumscribed circles of the second triangle, then will $\frac{1}{4} r^2 = pp'$.

16. The squares of the tangents from any point to a parabola are to one another as the focal distances of the points of contact.

17. S and H are foci, and C the centre of an ellipse. SM, HN are perpendiculars on the normal at P; prove that $CM = CN = \frac{1}{2}(SP \sim HP)$.

Also, find the polar equation to the locus of N.

18. A triangle is escribed about a parabola; prove that the area of the triangle whose vertices are the points of contact is double that of the escribed triangle.

19. The equation of a circle in which $(x_1\ y_1)$, $(x_2\ y_2)$ are ends of the chord of a segment containing an angle θ, is

$$(x-x_1)\ (x-x_2) + (y-y_1)\ (y-y_2)$$

$$\pm \cot\theta\left\{(x-x_1)(y-y_2) - (x-x_2)(y-y_1)\right\} = 0.$$

20. A parallelogram is described about an ellipse; if two of its angular points lie on the directrices, the other two lie on its auxiliary circle.