ARTS DEPARTMENT.

ARCHIBALD MACMURCHY, M.A., MATHEMATICAL EDITOR, *C. E. M.*

Our correspondents will please bear in mind, that the arranging of the matter for the printer is greatly facilitated when they kindly write out their contributions, intended for insertion, on one side of the paper ONLY, or so that each distinct answer or subject may admit of an easy separation from other matter without the necessity of having it re-written.

SOLUTIONS

by the proposer, J. H. BALDERSON, B.A., Math. Master, High School, Mount Forest.

201. Find the cube roots of unity and the factors of

x^2+x+1, $x^2\pm xy+y^2$, $x^3+y^3+z^3-3xyz$.

$x^3=1$ or $x^3-1=0$ or $(x-1)(x^2+x+1)=0$ we see 1 is a root, and by solving $x^2+x+1=0$ we find the other roots to be $\dfrac{-1\pm\sqrt{-3}}{2}$.

Now, if we let w be one root w^2 is the other; hence if 1, w, w^2, are the cube roots of unity, $w^3=w^6=w^9\ldots\ldots=1$, also, $w^2+w+1=0$ or $1=-(w+w^2)$.

Factors of x^2+x+1 are $(x-w)$, $(x-w^2)$.

Factors of x^2+xy+y^2 are $(x\mp wy)$, $(x\mp w^2y)$.

Factors of $(x^3+y^3+z^3-3xyz)$ are $(x+y+z)$, $(x+wy+w^2z)$, $(x+w^2y+wz)$.

202. If $X=ax+cy+bz$, $Y=cx+by+az$, $Z=bx+ay+cz$, then will

$$X^3+Y^3+Z^3-3XYZ =(a^3+b^3+c^3-3abc)(x^3+y^3+z^3-3xyz).$$

For $X^3+Y^3+Z^3-3XYZ$

$$=(X+Y+Z),(X+wY+w^2Z)\times(X+w^2Y+wZ).$$

$$a^3+b^3+c^3-3abc =(a+b+c)(a+wb+w^2c)(a+w^2b+wc).$$

$$x^3+y^3+z^3-3xyz =(x+y+z)(x+wy+w^2z)(x+w^2y+wz).$$

$$X+Y+Z=(x+y+z)(a+b+c).$$

$$X+wY+w^2Z=x(a+wc+w^2b) +y(c+wb+w^2a)+z(b+wa+w^2c),$$
$$=x(a+wc+w^2b)+yw^2(wc+w^2b+a) +zw(w^2b+a+wc),$$
$$=(x+w^2y+wz)(a+wc+w^2b).$$

Similarly

$$X+w^2Y+wZ =(x+wy+w^2z)(a+w^2c+wb),$$

$$\therefore\ (X+Y+Z)(X+wY+w^2Z)(X+w^2Y+wZ)=(x+y+z)(x+wy+w^2z)(x+w^2y+wz)(a+b+c)(a+wb+w^2c)(a+w^2b+wc),$$
$$=(x^3+y^3+z^3-3xyz)(a^3+b^3+c^3-3abc).$$

203. Prove that $\dfrac{(a+b+c)^5-a^5-b^5-c^5}{(a+b+c)^3-a^3-b^3-c^3}$

$$=\tfrac{5}{3}(a^2+b^2+c^2+ab+bc+ca).$$

$$(a+b+c)^3 =a^3+b^3+c^3+3(a+b)(b+c)(c+a);$$

$\therefore$ denominator becomes $3(a+b)(b+c)(c+a)$;

$\therefore$ the fraction is $\dfrac{(a+b+c)^5-a^5-b^5-c^5}{3(a+b)(b+c)(c+a)}$.

By taking the first two terms of the numerator, and the last two, we see that the numerator is divisible by $(b+c)$, and by symmetry it is divisible by $(c+a)(a+b)$; we see also that 5 is a factor by expanding the first term; $\therefore$ $(a+b+c)^5-a^5-b^5-c^5=5(a+b)(b+c)(c+a)$ (an expression of two dimensions); if we expand we find the type terms are Σa^4b, Σa^3b^2, Σa^3bc; also those of denominator are Σa^2b, Σabc; from those we see that a^2 is one of the terms of quotient, $\therefore$ b^2 and c^2; also we see that ab is one term,