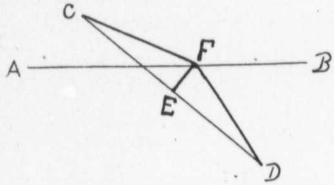
Euclid.

Deductions will be found to run in types. Below we give an interesting family of deductions fully solved. If you try to arrange deductions in this way for your class, you will find a great deal more interest will be secured.

I.

To find a point in a given straight line which is equally distant from two given points. Is this always possible?



Let AB be the given straight line, C and D the given points.

It is required to find in AB a point equally distant from C and D.

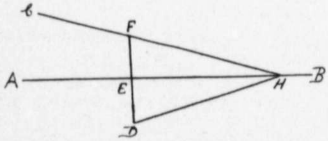
Join CD. Post. I.
Bisect CD in E. Prop. X.
From E draw EF perpendicular to CD, and meeting AB in F. Prop. XI.
Then F shall be the point required.
Join CF and BF. Post. I.
Then in the triangles EFC and EFD { EC = ED. Cons.
EF = EF.
∠CEF = ∠DEF. Def. X.
∴ the base CF = base DF. Prop. IV.

Wherefore a point F in the straight line AB has been found equally distant from C and D.

NOTE.—This is impossible if CD is perpendicular to AB, for then EF would be parallel to AB and would, therefore, never cut it.

II.

Two points are situated on opposite sides of a given straight line. Find a point in the straight line such that the straight lines joining it to the two given points may make equal angles with the given straight line. Is this always possible?



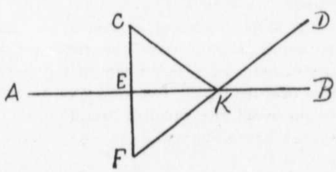
Let AB be the given straight line and C and D the given points. It is required to find in AB a point such that the straight lines joining it to the two given points may make equal angles with the given straight line.

From D draw DE perpendicular to AB. Prop. XII.
Produce DE to F. Post. II.
Cut off EF = DE. Prop. III.
Join CF. Post. I.
Produce CF to meet AB in H. Post. II.
Then H shall be the point required.
Join DH. Post. I.
Then in the triangles FEH and DEH { EF = ED. Cons.
EH = EH.
∠FEH = ∠DEH. Def. X.
∴ The angle FHE = angle DHE. Prop. IV.
Wherefore H is required point.

NOTE.—This would be impossible if C and D were equally distant from AB, for then CF would be parallel to AB, and would, therefore, never cut it.

III.

From two given points on the same side of a given straight line, show how to draw two straight lines which shall meet at a point in the given straight line and make equal angles with it.



Let AB be the given straight line and C and D the given points.

It is required to find a point in AB such that the two straight lines drawn from C and D to this point will make equal angles with AB.

Drop CE perpendicular to AB. Prop. XII.
Produce CE to F. Post. II.
Cut off EF = to EC. Prop. III.
Join FD cutting AB in K. Post. I.
Then K shall be the point required.
Join CK. Post. I.
Then in the triangles CEK and FEK { EC = EF. Cons.
EK = EK.
∠CEK = ∠FEK. Def. 10.
∴ the angle CKE = the angle FKE. Prop. IV.
But the angle FKE = the angle DKB. Prop. XV.
∴ the angle CKE = the angle DKB. Ax. I.
