a force equal to weight of $\frac{1}{8}$ triangle acting upwards through centre of gravity of OBC, the distance of centre of these from $A=\frac{13}{18}$ length of perpendicular from A on BC

$$=\frac{13}{18}\,3\sqrt{3}=\frac{13}{2\sqrt{3}} \text{ inches.}$$

5. A smooth inclined plane, whose height is one-half of its length, has a small pulley at the top, over which a string passes. To one end of the string is attached a mass of 12 lbs., which rests on the plane; while from the other end, which hangs vertically, is suspended a mass of 8 lbs.; and the masses are left free to move. Find the acceleration and the distance traversed from rest by either mass in 5 seconds.

The effective part of the force of 12 lbs. in retarding motion is 6 lbs.; the weight will move up the plane with an acceleration $f=\frac{8-6}{12+8}g$ per second $=\frac{g}{10}$; space traversed in 5 seconds $=\frac{1}{2}\cdot\frac{g}{10}\cdot 25=\frac{5g}{4}=40$ feet.

UNIVERSITY OF TORONTO.

ANNUAL EXAMINATIONS, 1881.

PROBLEMS (ALL THE YEARS).

Examiners — Charles Carpmael, M.A., A. K. Blackadar, B.A., F. Hayter, B.A.

1. If a point O be taken in the interior of an equiangular triangle ABC, and if we drop perpendiculars OH, OI, OL on the three sides, the sum of these three perpendiculars is equal to the altitude of the triangle.

(To be solved by geometry.)

2. Find θ, ϕ from the equations,

$$p\sin^4\theta - q\sin^4\phi = p,\quad p\cos^4\theta - q\cos^4\phi = q.$$

Investigate whether θ, ϕ can both be read for any real values of p and q.

3. If lines be drawn from the angles of a triangle ABC to the centre of the inscribed circle cutting the circumference in D, E, F, shew that the angles DEF of the triangle formed by joining these points are respectively equal to

$$\frac{\pi+A}{4},\ \frac{\pi+B}{4} \text{ and } \frac{\pi+C}{4}.$$

4. Let $a_1\, a_2\, a_3 \ldots$ be the lengths of the sides of a polygon $ABCD \ldots$ inscribed in a circle, $p_1\, p_2 \ldots$ the lengths of the perpendiculars from any point P in the circle on the considered position. Then if the polygon be not reëntering and if P be on the smaller arc cut off by a_1,

$$\frac{a_1}{p_1}=\frac{a_2}{p_2}+\frac{a_3}{p_3}+\ldots\ldots+\frac{a_n}{p_n}.$$

5. Prove that

$$\tan\frac{\pi}{2^{n+1}}\left\{\tan\frac{\pi}{2^{n+1}}+2\tan\frac{\pi}{2^n}+\ldots\right.$$
$$\left.+2^{n-2}\tan\frac{\pi}{2^3}+2^{n-1}\right\}=1.$$

6. If $\frac{bx+cy+az}{cx+ay+bz}=1$, shew that

$$\frac{b-c}{cy-bz}=\text{anal.}=\text{anal.}$$

7. Solve $\left.\begin{aligned} x^2-yz&=a \\ y^2-zx&=b \\ z^2-xy&=c \end{aligned}\right\}$

8. If n be any integer >1, shew that

$$\frac{\lfloor 2n}{\lfloor n} < \left\{8n^2(2n^2-1)\right\}^{\frac{n}{3}}$$

9. Examine the statement that every even number is the sum of two prime numbers, and every odd number the sum of three prime numbers.

10. Sum the series

$$\frac{1^2}{\lfloor 2}+\frac{2^2}{\lfloor 4}+\frac{3^2}{\lfloor 6}+\ldots\ldots \text{to infinity.}$$

$$\tan^{-1}\frac{4}{1.5}+\tan^{-1}\frac{6}{5.11}+\tan^{-1}\frac{8}{11.19}+\ldots\ldots$$

to n terms.

11. P, Q, R, S, are the middle points of the sides of a quadrilateral taken in order; the intersection of PR and QS lies in the same straight line with the points which bisect the diagonals of the quadrilateral.